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CONVECTIVE OVERSHOOT MIXING IN OLD OPEN CLUSTERS

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ABSTRACT

A simplified form of overshoot mixing of convective cores in intermediate-mass stars (0.90–1.40 $\mathfrak{M}_\odot$) is investigated in order to explain the anomalous gap structure of M67. The amount of overshooting is assumed to be proportional to the pressure scale height at the convective core boundary, and the entire region is completely mixed. Evolutionary tracks are computed for both standard and overshoot models, and isochrones are constructed. In the presence of overshoot mixing, the isochrones show (1) a higher gap, (2) a lower stellar number density immediately above the gap, and (3) the persistence of the gap to greater ages.

Subject headings: convection — interiors, stellar — open clusters — stellar evolution

Attempts to fit open clusters with theoretical isochrones (Sandage and Eggen 1969; McClure, Forrester, and Gibson 1974) have been moderately successful with the notable exception of M67 (Racine 1971; Demarque 1973). Torres-Peimbert (1971) has examined M67 with respect to standard models and has found that (i) the gap in M67 is much too high relative to the most luminous K subgiants in the cluster and (ii) the density of stars immediately above the gap is much lower than expected. Since M67 is the only well-observed representative of its age group, we shall assume that these properties are typical of an old open cluster that shows a well-defined gap. We must look for an effect which is maximal for masses characteristic of the stars near the M67 turnoff, but which is negligible for more massive or less massive stars.

This paper examines the result of a convective overshoot mixing proportional to the pressure scale height at the edge of the convective core as postulated by Roxburgh (1965). Although Saslaw and Schwarzschild (1965) have shown that energy transport by convective overshooting in the deep interiors is negligible, recent work by Shaviv and Salpeter (1973) demonstrates that the time-average amount of such overshooting may be significant. We shall ignore any energy transport by the overshooting elements, but shall be concerned only with the mixing beyond the core boundary which is affected by it. This overshoot mixing should be thought of as random overshooting elements which mix hydrogen-rich material into the hydrogen-depleted core.

This type of mixing, as is pointed out by Shaviv and Salpeter (1973), is effectively realized only over a time scale much longer than the convective time scale of the core. The greatest amount of such mixing will occur in stars with convective cores with low masses which have longer nuclear time scales. Higher-mass stars will be affected less for two reasons. (i) The shorter main-sequence lifetimes will reduce the total number of overshooting elements and thus the amount of mixing. (ii) The total amount of material within a pressure scale height above the convective boundary is roughly constant. Hence, an overshoot mixing proportional to the pressure scale height will have a proportionally lesser effect on stars with larger convective cores. The lower-mass stars with no convective cores will not be affected.

The stellar models were calculated with the Yale evolution code which was revised in order to calculate

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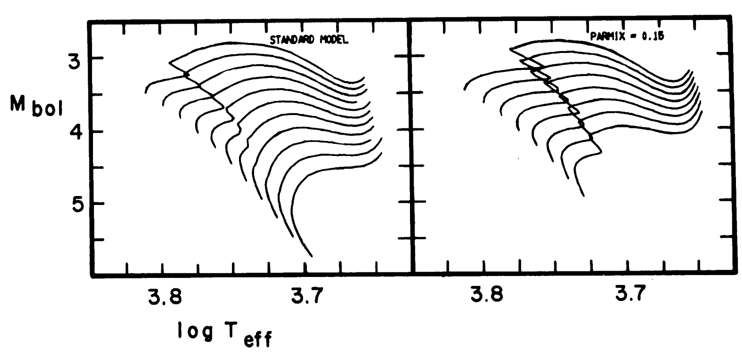

FIG. 1.—Evolutionary tracks in the (M_{bol}, log T_{eff})-plane. The standard sequence of tracks for masses 0.90–1.40 $\mathfrak{M}_\odot$ is on the left; the overshoot-mixed models with P_{mix} = 0.15 for masses 1.05–1.40 $\mathfrak{M}_\odot$, on the right.

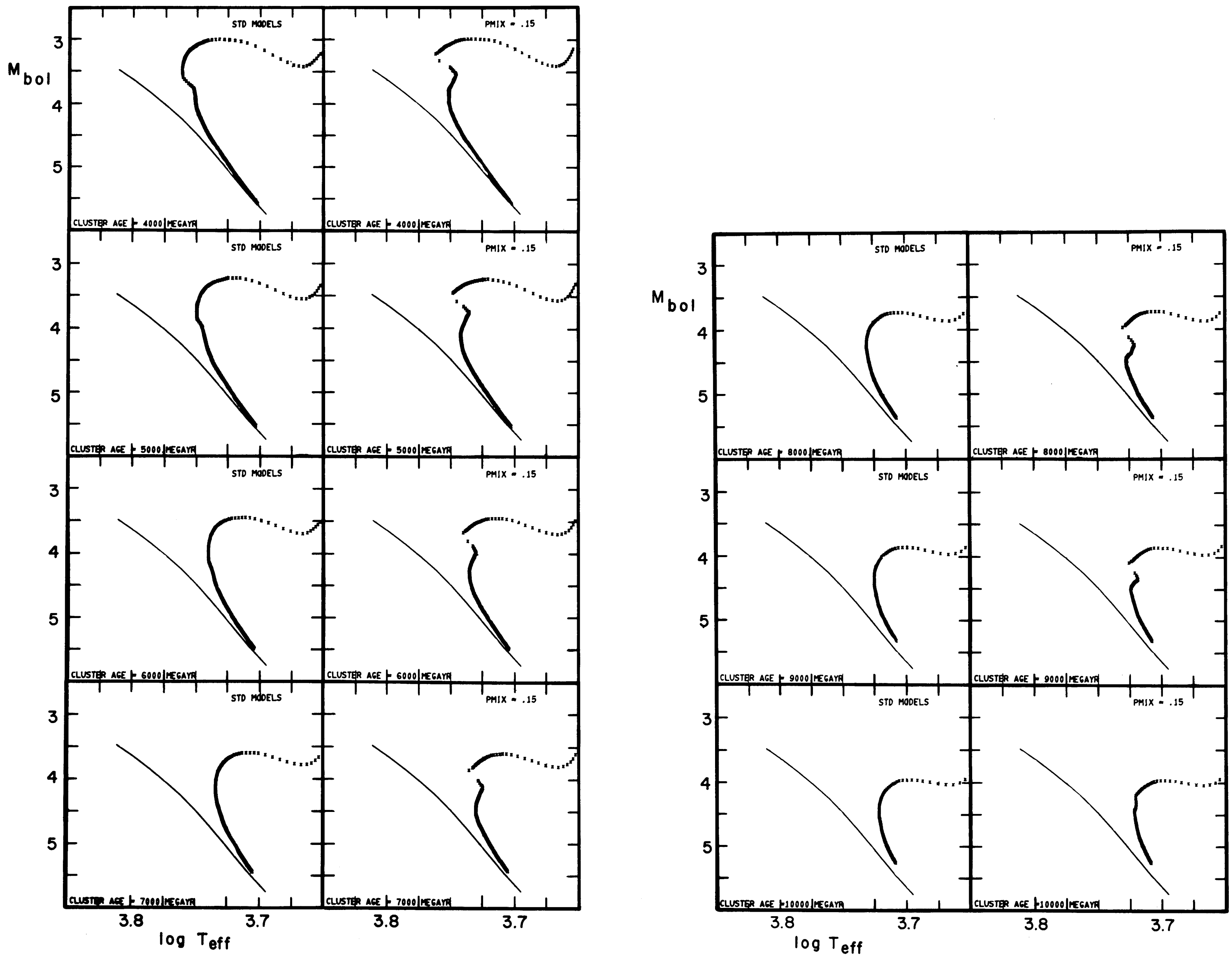

FIG. 2.—Isochrones for 250 stars (×) in the (M_{bol}, log T_{eff})-plane. The standard models (STD MODELS) are on the left; the overshoot-mixed models (PMIX = 0.15), on the right. The cluster ages, given in the lower-left corner, range from 4000 × 10^6 to 10,000 × 10^6 years in intervals of 1000 × 10^6 years. The ZAMS is denoted by a solid line.

the overshoot mixing. Assuming that the extent of the overshoot mixing is a function of the pressure scale height H at the edge of the convective core ($\delta R = P_{\text{mix}} H_c$), the mass which overshoots $\delta\mathfrak{M}$ was approximated in the program by

$$\delta\mathfrak{M} = 4\pi R_c^2 P_{\text{mix}} H_c \rho_c \,.$$

The subscript c denotes values at the edge of the convective core, and P_{mix} is the overshoot mixing parameter. The ratio of overshooting $(1 + \delta\mathfrak{M}/\mathfrak{M}_c)$ is then stored. In order to aid convergence of the models and to attempt to simulate the time-average aspect of the overshoot mixing, the mixing value for one model is applied to the subsequent one if the convective core still exists. Thus a star with a convective core for only one model is not affected. To avoid introducing additional parameters, the models were completely mixed within the overshoot region and the amount of mixing was independent of the time step between successive models. These latter two assumptions introduce an oversimplification of the model, unrealistic since the method of overshoot mixing postulated should produce a smooth composition gradient beyond the edge of the convective core (Maeder 1973) and should have a lesser effect for shorter time steps.

A standard set of tracks with no overshoot mixing was computed for 11 stars in the mass range $\mathfrak{M} = 0.90\,(0.05)\,1.40\,\mathfrak{M}_\odot$ with composition $(X, Y, Z) = (0.73, 0.25, 0.02)$. The opacities were taken from the tables of Cox and Stewart (1970) with the autoionization modifications of Watson (1970). The models were evolved until they started to turn up the giant branch at approximately $\log T_{\text{eff}} = 3.65$. The tracks were recomputed for $P_{\text{mix}} = 0.05$ and $P_{\text{mix}} = 0.15$ with the exception of the 0.09, 0.95, and $1.00\,\mathfrak{M}_\odot$ tracks. The latter is unaffected because it has a convective core for only one model, and the former two have none.

Isochrones for the standard and overshoot tracks are calculated by interpolating intermediate tracks. The computed tracks are divided arbitrarily into five segments. The break points in the tracks are defined for (*a*) tracks showing core exhaustion and (*b*) tracks without core exhaustion: (i) zero-age main sequence (ZAMS); (ii*a*) minimum $\log T_{\text{eff}}$ before core exhaustion; (ii*b*) maximum $\log T_{\text{eff}}$; (iii*a*) maximum $\log T_{\text{eff}}$ after exhaustion; (iii*b*) $X_{\text{core}} = 0$; (iv) maximum M_{bol}; (v) minimum M_{bol}; (vi) $\log T_{\text{eff}} = 3.650$. Between each pair of computed tracks, 25 intermediate tracks spaced linearly in M_{bol} along the ZAMS are calculated. This interpolation is done linearly ($\log T_{\text{eff}}$, M_{bol}) segment by segment. The corresponding age t for each track point is interpolated linearly in $t(L_0/\mathfrak{M})$, where $\mathfrak{M}$ is the mass and L_0 is the ZAMS luminosity. The resultant 250 tracks are then used to give isochrones for a given age by linearly interpolating in ($\log T_{\text{eff}}$, M_{bol}) versus time for each track. The isochrones for overshoot mixing are not very smooth in the range 1.00–1.05 $\mathfrak{M}_\odot$ because the morphology of the two neighboring tracks is quite different. The former track is unaffected while the latter is greatly influenced by the overshoot mixing.

The standard evolutionary tracks and those with $P_{\text{mix}} = 0.15$ are shown in figure 1. As expected, the main-sequence lifetimes of the overshoot-mixed models are extended with a corresponding reduction in the post–core-exhaustion lifetimes. The resultant effect on the isochrones is shown in figure 2. The spacing of the $\times$'s in figure 2, obtained from the 250 interpolated tracks, illustrates the relative density of stars in the H-R diagram of a cluster with a constant initial mass function (Schlesinger 1969). In the overshoot-mixed models, the gap is much higher with respect to the subgiants, the density of stars immediately after the gap is reduced, and the gap persists to much greater ages.

A comparison of the isochrones for $P_{\text{mix}} = 0.15$ and $P_{\text{mix}} = 0.05$ (not shown) indicates that a P_{mix} of the order of 0.1 would give good agreement with the height of the gap observed in M67. Thus, it is possible to explain the anomalous gap structure of M67 with the assumption of a convective core overshooting which results in mixing beyond the boundary of the convective core. Isochrones constructed under this assumption with the overshooting proportional to the pressure scale height show qualitative agreement with M67, with respect to both the height of the gap and the stellar luminosity function above the gap.

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