4

TEACHING MANUAL

Raffles and fun

WINTER 2019 MEETING 4

JANUARY 29 - 30

Contents

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2019 UCI MATH CEO COMMUNITY EDUCATIONAL OUTREACH.
UNIVERSITY OF CALIFORNIA AT IRVINE

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Meeting 4: Raffles and Fun

- Tuesday 9:00 AM - 9:50 AM
  - Place: **UCI** NS 2 1201 (Marco Forester comes)
- Tuesday 2:45 PM - 3:45 PM
  - Place: **SANTA ANA**: Carr Intermediate School
- Wednesday: 2:00 PM - 3:45 PM
  - Place 1: **UCI,** NS2 1201 (Lathrop comes)
  - Place 2: **UCI,** PSCB 140 : (Villa comes)

<table>
<tr>
<td>Tuesday Morning
(50 minutes)
January 29

• Crash course: 8:45 - 9:00 in the same room

• Activity 1: 45 minutes
• Weekly Youth Survey: 5 minutes</td>
<td rowspan="2">Wednesday Afternoon
(80 minutes)
January 30

• Activity 1: 40 minutes
• Activity 2: 20 minutes
• Stock market Game: 20 minutes
• Weekly Youth Survey: 5 minutes
Start at 3:35</td>
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<tr>
<td>Tuesday Afternoon
(50 minutes)
January 29

• Activity 1: 45 minutes
• Weekly Youth Survey: 5 minutes</td>
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ACTIVITY 1: LOTTERY

Time: 40 minutes

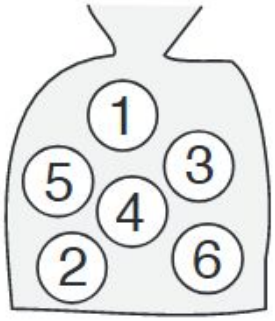
LOTTERY

Karl is thinking of holding a mini lottery to raise money.

I will sell tickets like this for $1 each.

1	2	3
4	5	6

Each player must put a cross through 2 numbers on the ticket and hand it in. At the end of the week I will draw out two balls from a bag.



Every player who has chosen the same two numbers as shown on the balls will win a cash prize of $10.

1. How many ways are there of choosing two different numbers on the ticket? Show all your working.

2. Will the lottery be a good money raiser? Describe your reasoning.

Experimental Probability is what actually happens when we try it out.

Step 1: Cross out two numbers on your ticket.

1	2	3
4	5	6

Step 2: Student 1 picks two balls from the raffle bag and calls out the two numbers. (Do not put the first ball back in the raffle bag.) Everybody will record the numbers on their chart below. Put the 2 balls back into the raffle bag. Student 2 picks two balls from the raffle bag and calls out the two numbers. Everybody records, … Your team will pick and record 30 times.

Step 3: My two numbers are _____ and _____.

My experimental probability of my two numbers being picked is _____ out of 30.

1	2	3
4	5	6

How many ways are there of choosing two different numbers on the ticket?

Will the lottery be a good money raiser? Describe your reasoning.

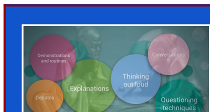
Solution and reasoning for 1	Solution and reasoning for 2

ACTIVITY 1: LOTTERY	
Description	In this task, students will count pairs of numbers from a certain set, using different counting strategies. They will need to make sense of whether the order of the numbers matters or not, in the context of the problem. Then, based on the result, and the data in the problem, they need to reason whether certain lottery plan is a good opportunity to raise money, or if on the other hand, it does not seem to provide much money at all.
Learning Goals	● I can count pairs of values from a given list and visualize all the options. ● I can evaluate how good is a plan in financial (money) terms, based on numerical information.
Materials	● Student Workbooks ● Tokens or Ping pongs (to simulate the lottery)
Set-up	● Have students read the problem individually. ● Once this is done, ask one or more students to explain the problem **using their own words**. ○ Guide them to be precise in their explanation (but that does not mean using the same words as the statement, in fact, encourage students to use their own words). ● Encourage kids to work in groups of 2 or 3. If desired, and depending on your group, you may also do part of the activity all together, leading with questions. If that is the case, make sure to ask questions to *all* kids, and not just 1 or 2.
My solution	In this space, write your solution to the problem (working out details, not just the final answers). Use as many visual representations as possible! Also, write discussion questions: these are questions that help students, at the end, consolidate the math learning. **My solution** 

My discussion questions (some examples are included)

- How would this problem change if we had only five numbers 1-5? How about numbers 1-10? Explain your thoughts.
- ____
____
____
- ____
____
____

Productive discussion	This section gives you examples of prompts, cues and questions that you may ask students during or at the end of the problem solving process. Before you continue, please watch:



Communication in the Teaching and Learning of Math
More Math 192 Series Videos:
(www.math.uci.edu/mathceo/teachingvideos.php)

	• **If some groups are not able to "start" (overwhelmed)**
	  ○ *"Can you give some examples of pairs that you can build?"*
	  ■ It is important that students understand the objects that we are counting in the problem. They should generate plenty of examples.
	
	• **If you see two students who seem shy or are working in isolation**
	  ○ *"Hey Alan and Bianca, I see that you are working alone, maybe you want to work together for a while? I think you can learn a lot from each other"* "Melissa, I think you can give Nora great advice in this part!"
	  ■ Don't force them to pair up: instead, you should invite them to do so **and provide at least one reason for it**.
	
	• **If you see a student working in isolation who seems quite comfortable figuring out the problem**
	  ○ *"Linda, would you like to present (all or part) of your solution to these students and take questions from them" ; "I see that you have the answers, but it's also important that you can talk and convince others"*
	  ■ This can be especially useful to spark communication skills in **students who do not see themselves as "good communicators"** but are confident in math.
	
	• **Scaffolding / testing for understanding**
	  ○ *"Is 1,3 the same as 3,1? Why or why not? Convince me!"*
	  ■ It's important that students realize that they are choosing a set of 2 numbers, and so that the order does not matter.
	
	• **If you see a wrong solution**
	  ○ *"I'm curious why you got 30 options. You multiplied 6 times 5". Did you count taking order into account?*
	  ○ *"Why did you add 6 + 5? What is the meaning of adding when we count things?"*
	  ■ Notice the positive language, non-judgemental, but critical in a good way. It's important to inspect the process and not just say that the answer is wrong and correct it (which is tempting but will not result in meaningful learning from the student, since you will not reach the "source of the mistake").

Teaching tips	• It's always a good idea to start the activity with an informal chat with students about the situation that is presented.
	  ○ Tell students: Suppose that from this table of [N] students, we are going to choose 2 to go to a trip. Do you think that the order would matter? Give some example of choices. Do you think that there are more, or less than N choices?
	  ■ Illustrate that order does not really matter: for example, selecting Alan and Diana is the same as selecting Diana and Alan. Have students give some examples.
	  ○ This warm-up can help students to get engaged in the math.
	
	• This problem gives a good opportunity for students to understand the

	difference between the following operations when counting: ○ N + N-1 : we use + when counting separate things. ○ N x N : this would mean counting all numbers from 00 to 99, where order matters (N=10) ○ N x (N-1). : similar to the above, but without repeating digits. ○ (N x (N-1)) / 2. : counting pairs. Divide by two to reduce the objects, because for each object we were counting it twice. It is not important to go over these, but rather to bring them to conversations if you feel students need some guidance, which is perfectly fine. Always do a concrete example with 2 or 3 small values of N. ● When asking questions, make eye contact with students. Listen carefully, with undivided attention. They will care more if they see that you care for their thinking (not just their answers). ● Before starting, clarify the key terms involved in the problem: *pair, ticket, expected, prize, order does not matter*. You can provide sentence stems such as: ○ The pair _____ is the same as the pair _____ because the order does not matter in this problem. It does not matter because ___.

Solutions (1 LOTTERY)

1) *How many ways are there of choosing two different numbers on the ticket?*

Solution: there are 15 such ways. We can list them as follows:

- That contain 1: { 1 2 } { 1 3 } { 1 4 } { 1 5 } { 1 6 }
- That contain 2 (and not listed already): { 2 3 } { 2 4 } { 2 5 } { 2 6 }
- That contain 3 (and not listed already): { 3 4 } { 3 5 } { 3 6 }
- That contain 4 (and not listed already): { 4 5 } { 5 6 }
- That contain 5 (and not listed already): { 5 6 }

Another solution:

- To select the first value, there are 6 options.
- For the second value, there are now 5 options (1 less than before, as we already selected one value)
- Thus this gives 30 values. But note that each pair has been counted twice! (example: 2 and 5 is really the same as 5 and 2, but we are counting it twice). So we need to divide 30 by 2, which gives 15.

Note: although not required in the question, it is helpful to ask students to list the 15 solutions, or if they come out with an incorrect number, it is helpful to ask students to list the pairs, so that they realize which pairs were missing or being repeated.

2) *Will the lottery be a good money raiser? Describe your reasoning*.

Solution: It is a good money raiser (assuming that you have enough buyers).

Remember that each ticket costs $1, and that the mayor prize is $10. Each person picks two values from 1 to 6 to mark his ticket. So each person has odds of one out of 15 to win: P = 1/15 (6.6%). You may ask stu3

This means that, for example, if 15 people buy the lottery, then you expect 1 person to win it (and thus your earnings would be 15 minus 10 = $5). If 150 buy the lottery, you would expect 10 people to guess the pair.

The following table illustrates more cases:

# of people	**Income ($)**	**Expected money needed to pay ($)**	**Expected earnings ($)**
15	15	1 x 10 = 10	15 - 10 = 5

30	30	2 x 10 = 20	30 – 20 = 10
45	45	3 x 10 = 30	45 – 30 = 15
60	60	4 x 10 = 40	60 – 40 = 20

So every new 15 buyers increases the expected earnings by $5.

Note: note that the earnings in the table are expected, but not guaranteed. In fact, if for example, you have 15 buyers and somehow all of them guess the pair (which is extremely unlikely, but possible), you would have to pay $150, and thus you would actually lose $140! To account for that (and avoid this risk), you may change the rules of the lottery. For example:

- Tell participants that the big prize will now be $15, but that it will be split among all winners
- Have a chart that limits every pair to be chosen at most 2 times (or at most 3 times if you have more than 30 buyers, at most 4 times if you have more than 45 buyers, etc).

  You should encourage students to use the term “expected” in their vocabulary, and also to come up with these variations of the lottery.

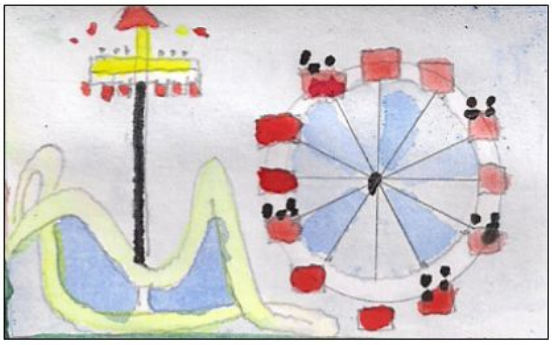
ACTIVITY 2: THE SANTA ANA AMUSEMENT PARK

Time: 20 - 30 minutes

2) THE SANTA ANA AMUSEMENT PARK

Welcome to the brand new Santa Ana Amusement Park!

Will you swim in the crazy SPLASH LAND, see wild animals in ADVENTURELAND, take a spooky tour in SPOOKYLAND or ride the Chicago wheel in WONDERLAND?



A group of five friends love the Amusement Park and plan to go there everyday after school during the year. They stop at the ticketing office to buy entrance tickets. There are four options:

i) Monthly Pass (30 entries)	$44
ii) Bi-weekly Pass (15 entries)	$21
iii) 10-Day Pass (10 entries)	$14.5
iv) Daily Pass (1 entry)	$1.5

Buying 30 entries

A Which one is the cheapest way to buy exactly 30 entries? (you may mix different options)

i) Monthly Pass (30 entries)	$44
ii) Bi-weekly Pass (15 entries)	$21
iii) 10-Day Pass (10 entries)	$14.5
iv) Daily Pass (1 entry)	$1.5

Buying 21 entries

B Which one is the cheapest way to buy exactly 21 entries? (you may mix different options)

Cheapest Fare

C Which of the 4 options gives the *cheapest fare per day per person*?

Annual Fare for 2

D The park has hired you to set an annual fee (actually 360 days) for a couple (2 people), such that its fare per day is strictly between 80% and 90% of the current cheapest fare per day. Come up with one, explaining your solution.

v) Annual Pass (2 entries) $_____

Remodeling

The Amusement park has four different sections (each with several attractions), all having equal size: Splash Zone, Wonderland, Spook Land, and Adventureland.

The group of friends were sad because some parts of each section of the park were closed due to remodeling: one sixth of *Splash Zones*, one third of *Wonderland*, thirteen twenty sixths (13/26) of *Spook Land* and four sixths of *Adventure Land* would be closed to the public.

E Help them figure out what fraction of the park was closed. Express your answer as a fraction.
Was more than 50% of the park closed?

ACTIVITY 2: THE SANTA ANA AMUSEMENT PARK	
Description	In this task, students deal with comparing unit fares (either using fractions, ratios or decimals) and come up with fares that fit a certain restriction. Later, they will need to compare fractions that are applied to different subtotals to determine a fraction of the total. This is all in the context of a theme park.
Materials	• Student workbook
Set-up	You may keep the same groups as before. But this time, allow for 3 minutes of individual work for question A).
My solution	In this space, write your solution to the problems (working out details, not just the final answers). Use as many visual representations as possible! Also, write discussion questions: these are questions that help students, at the end, consolidate the math learning. **My solution** 

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<td>
 My discussion questions (some examples are included)

In this problem, was it easier to work with ratios, with fractions or with decimals? Explain why.
____
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<td>Productive discussion</td>
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This section gives you examples of prompts, cues and questions that you may ask students during or at the end of the problem solving process.

If some groups are not able to “start” (overwhelmed)

“Suppose that you have $100”. How many days could you enter the park, buying each of the passes?

This can help you find out to what extent the students comprehend the situation, and go from there. They may not answer the question, but talk about how they would answer it.

If you see two students who seem shy or are working in isolation

“Hey Alan and Bianca, I see that you are working alone, maybe you want to work together for a while? I think you can learn a lot from each other”

At this point, you may choose to keep the same groups from the previous activity, but you may choose to form new groups strategically (to prevent some distractions, or to pair students of different mathematical backgrounds).

If you see a student working in isolation who seems quite comfortable figuring out the problem

“Linda, would you like to present (all or part) of your solution to these students and take questions from them” ; “I see that you have the answers,

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	but it's also important that you can talk and convince others". ○ *I see that you made a very nice diagram representing the different fares. Please share it!* ■ Insist on this, but still be very gentle about it. If the discussion in activity 1 was "dominated" by only a few students, focus on other students. Don't let anyone behind! ● **Scaffolding / testing for understanding** ○ *"Draw a table that represents money vs days, for the first pass"* ■ Although a table may not be the ideal way to solve the problem, using this strategy in the beginning may be helpful for students, so that they make sense of each pass. ● **If you see a wrong solution** ○ *"I'm curious why you got this value. Guide me through it! I want to understand what you were thinking. Is it OK to say that the pass with the smallest price tag is the best option?* ■ Notice the positive language, non-judgemental, but critical in a good way. It's important to inspect the process and not just say that the answer is wrong and correct it (which is tempting but will not result in meaningful learning from the student, since you will not reach the "source of the mistake".
Teaching tips	● This activity provides a good opportunity to introduce (or refresh) the concept of common multiple (or least common multiple), which is important when comparing rates that do not have the same denominator, such as in this problem. ● For Part A, try to scaffold your students by prompting them with the questions: "What do we want to know? What do we need to know to figure that out?" ○ Students may respond in various ways. For example, one student may say that they have no idea. For these students you may ask them to reread the question, and ask about some language you may suspect they are unsure of. Other students may immediately answer with the information they need to figure out. Let these students move on and encourage them to help others! ● For Part D, you may want to chunk the question by introducing the question in pieces. Depending on your group you may do this in a variety of ways. For example, you may ask your students to read a sentence and then pick another student to read. ● For Part E, it may also help to chunk the information given in the problem. Ask your students to reorganize the situation in a way that makes sense to them, whether that is a list, graphic organizer, table, graph, picture, or something else. Once they do that, ask them to explain their thoughts verbally.

Solutions (2 THE SANTA ANA AMUSEMENT PARK)

A) Which one is the cheapest way to buy exactly 30 entries? (you may mix different options).

Solution: Note that all times periods are multiple of 30. We find each fare:

		Total ($) for 30 entries
Monthly:	1 x (30 entries)	44 x 1 = 44
Bi-weekly:	2 x (15 entries)	21 x 2 = 42
10-day:	3 x (10 entries)	14.5 x 3 = 43.5
Daily:	30 x (1 entry)	1.5 x 30 = 45

So buying 2 Bi-weekly passes gives the cheapest way.

B) This is interesting, as there are different ways to come up with 21 entries by mixing different options, and they cannot use just A, because the Bi-weekly pass is for 15 days and 15 is not a factor of 21.

Different ways or combinations:

Combination	Cost ($)
21 x (1 entry)	31.5
2 x (10 entries) + 1 (1 entry)	30.5
1 x (15 entries) + 6 (1 entry)	30

So the cheapest way is to get 1 by-weekly pass and 6 daily passes.

C) We can use A). Here is another way:
We first find each daily cost (per person) by creating the fraction Price / # of days:

Ticket type	**Price ($)**	**# of days**	**Daily cost ($)**
Monthly	44	30	44/30 (or 22/15)
Bi-weekly	21	15	21/15 (or 7/5)
10-day	14.5	10	14.5/10 (or 1.45)
Daily	1.5	1	1.5

Note that 21/15 < 14.5/10 < 44/30 < 1.5. We justify this as follows:

- 21/15 < 14.5/10 because 6/15 < 4.5/10. Note that 6/15 < 4.5/10, because 6/15 = 2/5 = 4/10 = 0.4 (and 0.4 is less than 0.45).
    - Here we used: to compare numbers larger than 1, we can remove 1 from both

and do the comparison (for example, 1.6 < 1.7 because 0.6 < 0.7). This is helpful with fractions.

- 14.5/10 < 22/15 because 4.5/10 < 7/15. Note that $\frac{4.5}{10} < \frac{7}{15}$, because 67.5 < 70.
- 22/15 < 1.5 because 7/15 < 0.5, because 7/15 < 7/14 = 0.5.

Thus, the bi-weekly pass is the cheapest (7/5).

D) *The park has hired you to set an annual fee (actually 360 days) for a couple (2 people), such that its fare per day is between 80% and 90% of the current cheapest fare per day.*

Solution: The cheapest fare is $7/5 or $1.4. Note that:

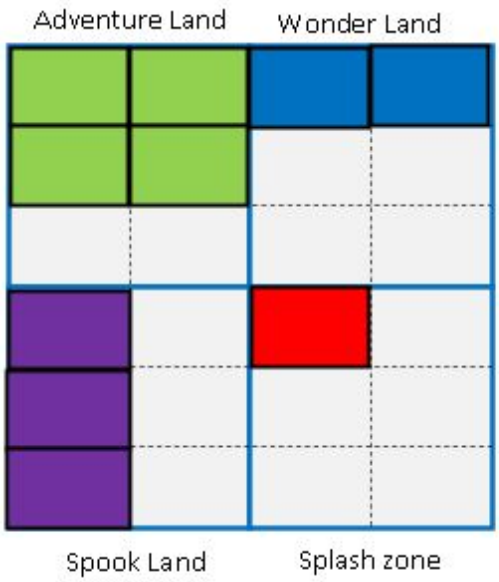
- 10% of 1.4 is 0.14
- 80% of 1.4 is then 0.8 + 0.32 = 1.12
- 90% of 1.4 is 1.4 minus 0.14, which is 1.26
- So we need a price between 1.12 and 1.26. For example, 1.2 would do.

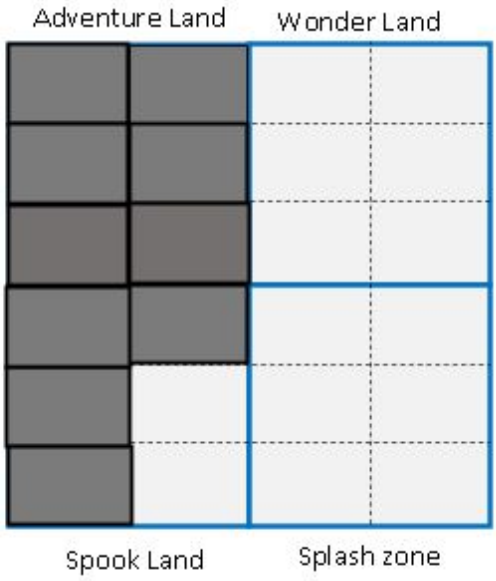
However, 1.2 is a price for 1 day, for 1 person, and we need to give a fare for a 360 fee for 2 people. So what we need to do is multiply 1.2 by 360 and then by 2.

One way: "look for a 9":
1.2 x 360 x 2 = 12 x 36 x 2 = 4 x 3 x 36 x 2 = 8 x 3 x 36 = 16 x 3 x 18 = 32 x 3 x 9 = 96 x 9 = 960 - 96 = **864**.
So we should charge $864, or even $865 (it can be easily seen that this will still be below 1.26 daily per person).

E) *Figure out what fraction of the park was closed. Express your answer as a fraction.*

Solution:



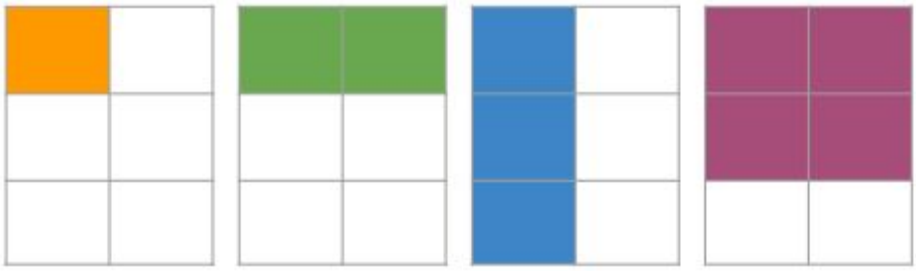


We find the fraction of each closed part. Note that this involves multiplying by 1/4 each time, since all parts have the same size and there are 4 of them.

(1/6) x (1/4) = 1/24
(1/3) x (1/4) = 1/12 = 2/24
(13/26) x (1/4) = 1/8 = 3/24
(4/6) x (1/4) = 1/6 = 4/24

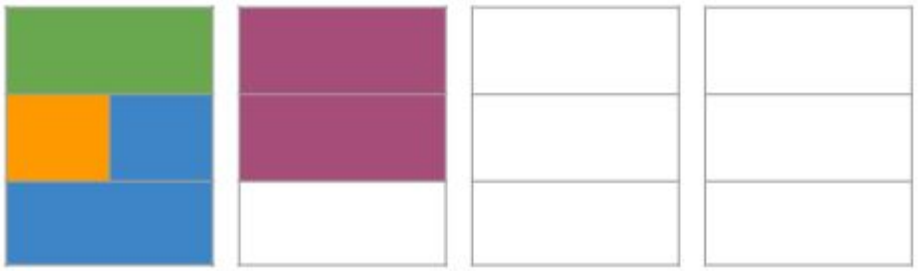
So the TOTAL is: 10/24 (or 5/12). This is less than 50%.

Another solution: First we add all the closed parts: so we add the fractions, all expressed in terms of sixths to simplify calculations:



1/6 + 2/6 + 3/6 + 4/6 = 10/6 = 5/3.

Note that since all four attractions are equal in size, we can think that they have the same "weight" (of 1), and so we can think of the park having a total weight of 4. This means that we have to divide 5/3 by 4:



(5/3) / 4 = 5/12. This is less than 50%.