2

MATH CEO

Competition!

WINTER 2019 MEETING 2

JANUARY 15 - 16

Contents

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2019 UCI MATH CEO COMMUNITY EDUCATIONAL OUTREACH.
UNIVERSITY OF CALIFORNIA AT IRVINE

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Meeting 2: Competition!

- Tuesday 9:00 AM - 9:50 AM
  - Place: **UCI** NS 2 1201 (Marco Forester comes)
- Tuesday 2:45 PM - 3:45 PM
  - Place: **SANTA ANA**: Carr Intermediate School
- Wednesday: 2:00 PM - 2:45 PM
  - Place 1: **UCI,** NS2 1201 (Lathrop comes)
  - Place 2: **UCI,** PSLC 1400 : (Villa comes)

<table>
<tr>
<td>Tuesday Morning
(50 minutes)
January 15

• Crash course: 8:45 - 9:00 in the same room

• Activity 1: 45 minutes
• Weekly Youth Survey: 5 minutes</td>
<td rowspan="2">Wednesday Afternoon
January 16
(80 minutes)

• Activity 1: 40 minutes
• Activity 2: 20 minutes
• Stock market Game: 20 minutes
• Weekly Youth Survey: 5 minutes
Start at 3:35</td>
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<tr>
<td>Tuesday Afternoon
(50 minutes)
January 15

• Activity 1: 45 minutes
• Weekly Youth Survey: 5 minutes</td>
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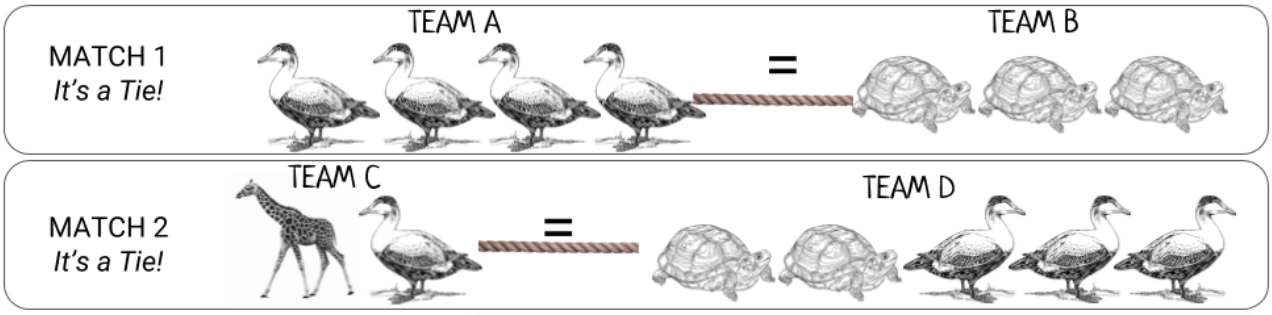
ACTIVITY 1: TUG OF WAR

Time: 20 - 30 minutes

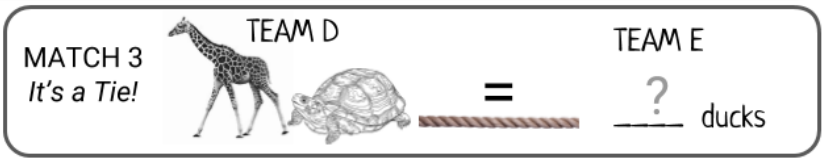
1 Tug of War

Directions:
1) You and your partner will each build Match 1 & Match 2 on your own whiteboards.
2) Whiteboard 1 will be used as a guide to help you solve all the problems.
You and your partner will **not** move the animals on this whiteboard.
3) Use Whiteboard 2 to solve the problems. You and your partner will move, add or remove the animals on this whiteboard to solve the problems.

Ducks, turtles and baby giraffes play tug of war in the zoo. The first two matches are a tie, which means that opposite teams are pulling the rope with the same force. It's a perfect balance!



A) Match 3
How many ducks do you need to balance a team of a baby giraffe and a turtle?

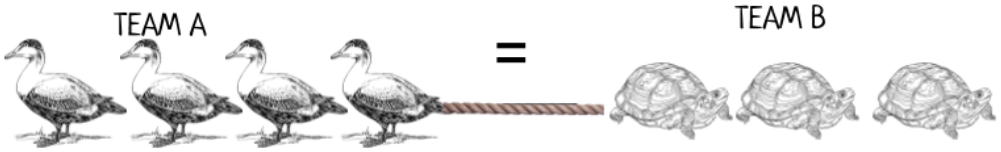


B) Match 4
How many turtles do you need to balance a team of 2 baby giraffes?

Tug of War

C) Recall that the first two matches end in a tie:

MATCH 1
It's a Tie!



MATCH 2
It's a Tie!

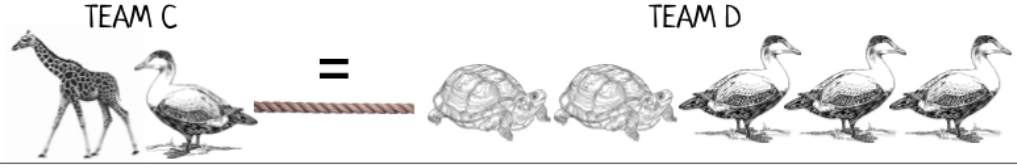
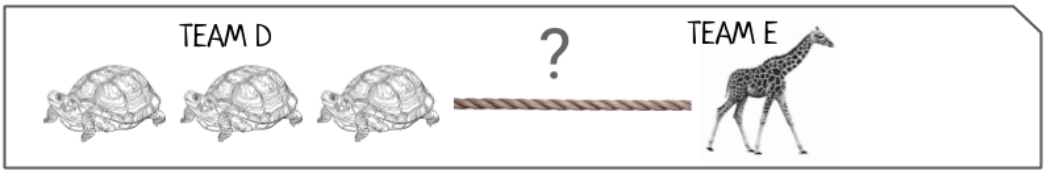
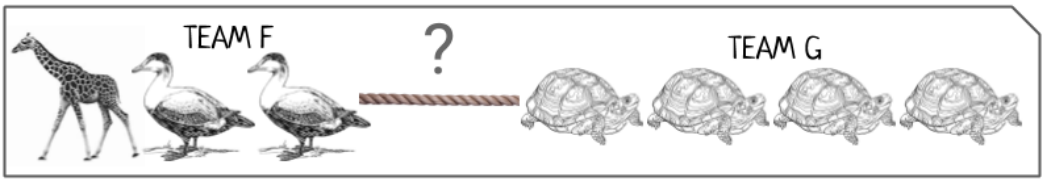


Figure out which team would win the fifth and sixth pulling competitions below. Explain why.
Note: you may also use the results in parts A and B.

MATCH 5
Who wins?



MATCH 6
Who wins?



ACTIVITY 1: TUG OF WAR	
Description	In this task, students encounter visual representations of equations (balanced situations), in terms of different animals pulling a rope, in two teams. Students need to make sense of these situations and be able to produce new information from given information. They may or may not use formal language in the process of solving these problems.
Materials	• Post-it notes • Animal tokens
Set-up	Let students think of each problem individually to come up with a solution plan.
My solution	In this space, write your solution to the problems (working out details, not just the final answers). Use as many visual representations as possible! Also, write discussion questions: these are questions that help students, at the end, consolidate the math learning. **My solution** 

My discussion questions (some examples are included)

- What are some ways in which we can create new "tied" situations, from the original ones? Explain.
- ____
  ____
  ____
- ____
  ____
  ____

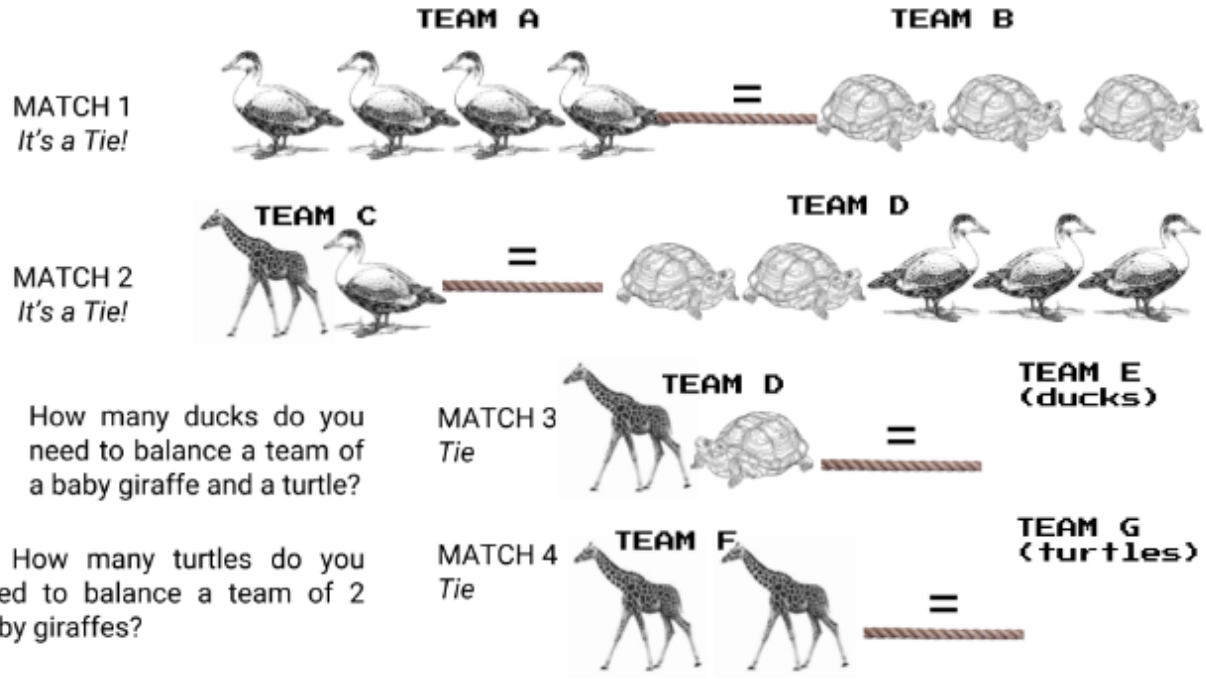
Productive discussion

This section gives you examples of prompts, cues and questions that you may ask students during or at the end of the problem solving process.

- **If some groups are not able to "start" (overwhelmed)**
  - *"Describe the picture. What do you see? What do you notice?*
    - This can help you find out to what extent the students comprehend the situation, and go from there.
- **If you see two students who seem shy or are working in isolation**
  - *"Hey Alan and Bianca, I see that you are working alone, maybe you want to work together for a while? I think you can learn a lot from each other"*
    - At this point, you may choose to keep the same groups from the previous activity, but you may choose to form new groups strategically (to prevent some distractions, or to pair students of different mathematical backgrounds).
- **If you see a student working in isolation who seems quite comfortable figuring out the problem**
  - *"Linda, would you like to present (all or part) of your solution to these students and take questions from them" ; "I see that you have the answers, but it's also important that you can talk and convince others".*
  - *I see that you made a very nice diagram representing the balance. Please share it!*
    - Insist on this, but still be very gentle about it. If the discussion in activity 1 was "dominated" by only a few students, focus on other

	students. Don't let anyone behind! ● **Scaffolding / testing for understanding** ○ *"Come up with some values of 'forces' of each animal that would balance the first picture"* ■ Although trial and error may not be the ideal way to solve the problem, using this strategy in the beginning may be helpful for students. ● **If you see a wrong solution** ○ *"I'm curious why you got this value. Guide me through it! I want to understand what you were thinking. Do all animals of the same type apply the same force? Can you please check?"* ○ *"Why are you saying that this force is equal to [a specific value]]? I would like to understand"* ■ Notice the positive language, non-judgemental, but critical in a good way. It's important to inspect the process and not just say that the answer is wrong and correct it (which is tempting but will not result in meaningful learning from the student, since you will not reach the "source of the mistake".
Teaching tips	● It is important to make sure that kids can describe the balanced equations in their own words, using terms such as "equivalent", "equal", "same force", "same strength", "tied", etc. Encourage students to use this terms. This will be important for them to reason about the problems. ● Some students may want to abbreviate the visuals, using numbers or symbols. This should be encouraged. For example, "4 T" is equal to "3 T". However, make sure that whichever moves they use to manipulate these expressions are well understood in terms of the context of the problem, and that moreover they can justify them in words. ● It may be useful to remind students that all turtles have the same strength, all ducks have the same strength, etc. Although this is not explicit in the problem, it is implicitly so and students should make this assumption. If they ask you, you may have a short conversation telling them that sometimes in math and problems we make simplifying assumptions that may not hold true in real life but can help us reason through.

Solutions (1 TUG OF WAR)



A) Solutions

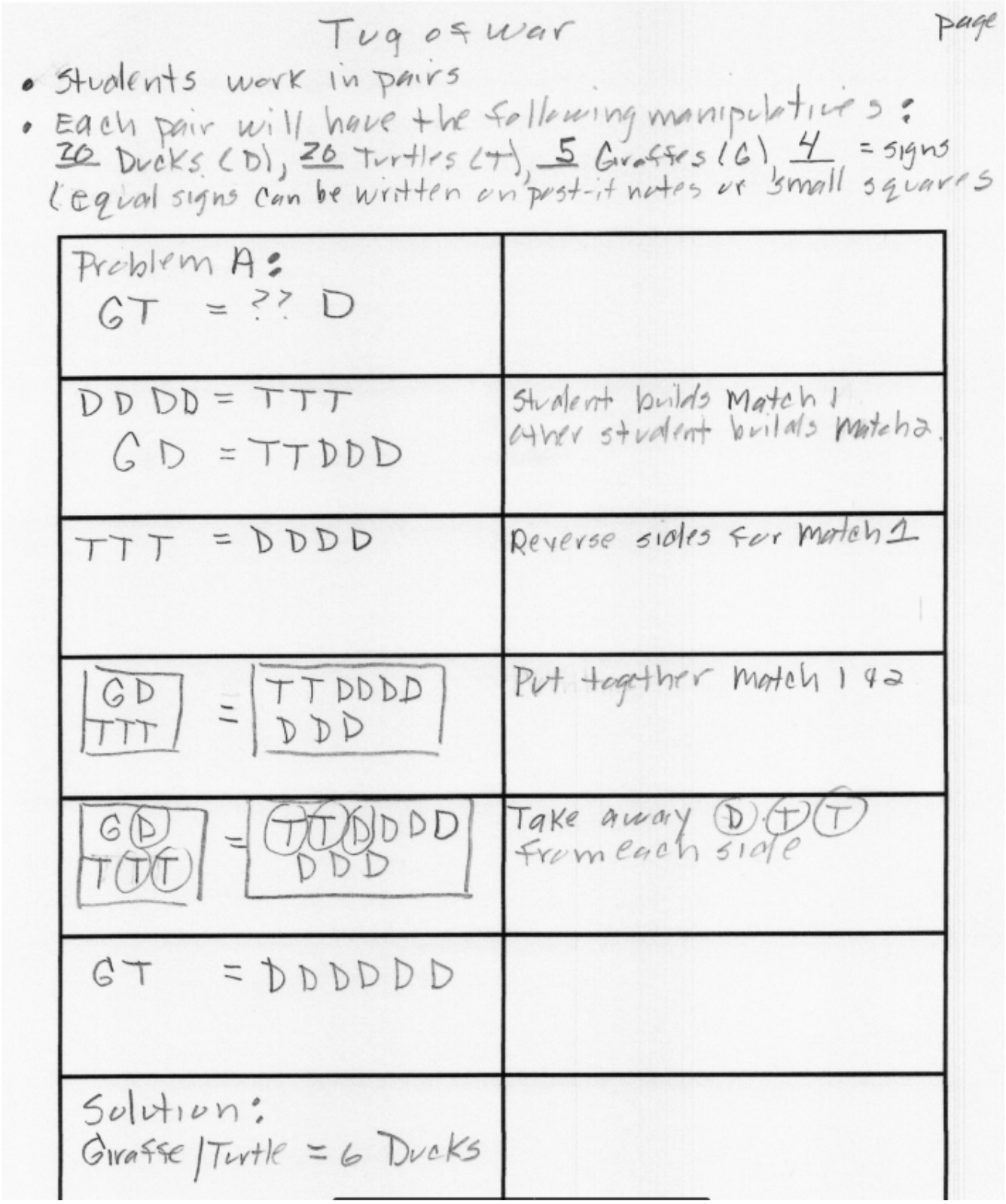
We know (matching teams) that "4 ducks plus 3 ducks and 2 turtles" have the same strength as "1 giraffe, 1 duck and 3 turtles". So:

- "7 ducks and 2 turtles" are equal in strength to "1 duck, 3 turtles and 1 giraffe".

Removing the same animals (and quantities) in each team will preserve the balance (you may ask students why they think that is true, in their own words). So if we remove 2 turtles and 1 duck:

- "**6 ducks**" are equal in strength to "1 turtle and 1 giraffe".

 Another solution:



Tug of War

page

- Students work in pairs
- Each pair will have the following manipulatives: 20 Ducks (D), 20 Turtles (T), 5 Giraffes (G) 4 = signs (equal signs can be written on post-it notes or small squares

Problem A: GT = ?? D	
DDDD = TTT GD = TTDDD	Student builds Match 1 other student builds Match 2.
TTT = DDDD	Reverse sides for Match 1
GD / TTT = TTDDDD / DDD	Put together Match 1 & 2
GDT / TTT = TTDDDD / DDD	Take away D T T from each side
GT = DDDDDD	
Solution: Giraffe / Turtle = 6 Ducks	

So the answer is: 6 ducks.

B) Solutions

By part A, we know:

- “12 ducks” are equal in strength to “2 turtles and 2 giraffes”.

But 12 ducks have the same strength as 9 turtles (by looking at match 1). So:

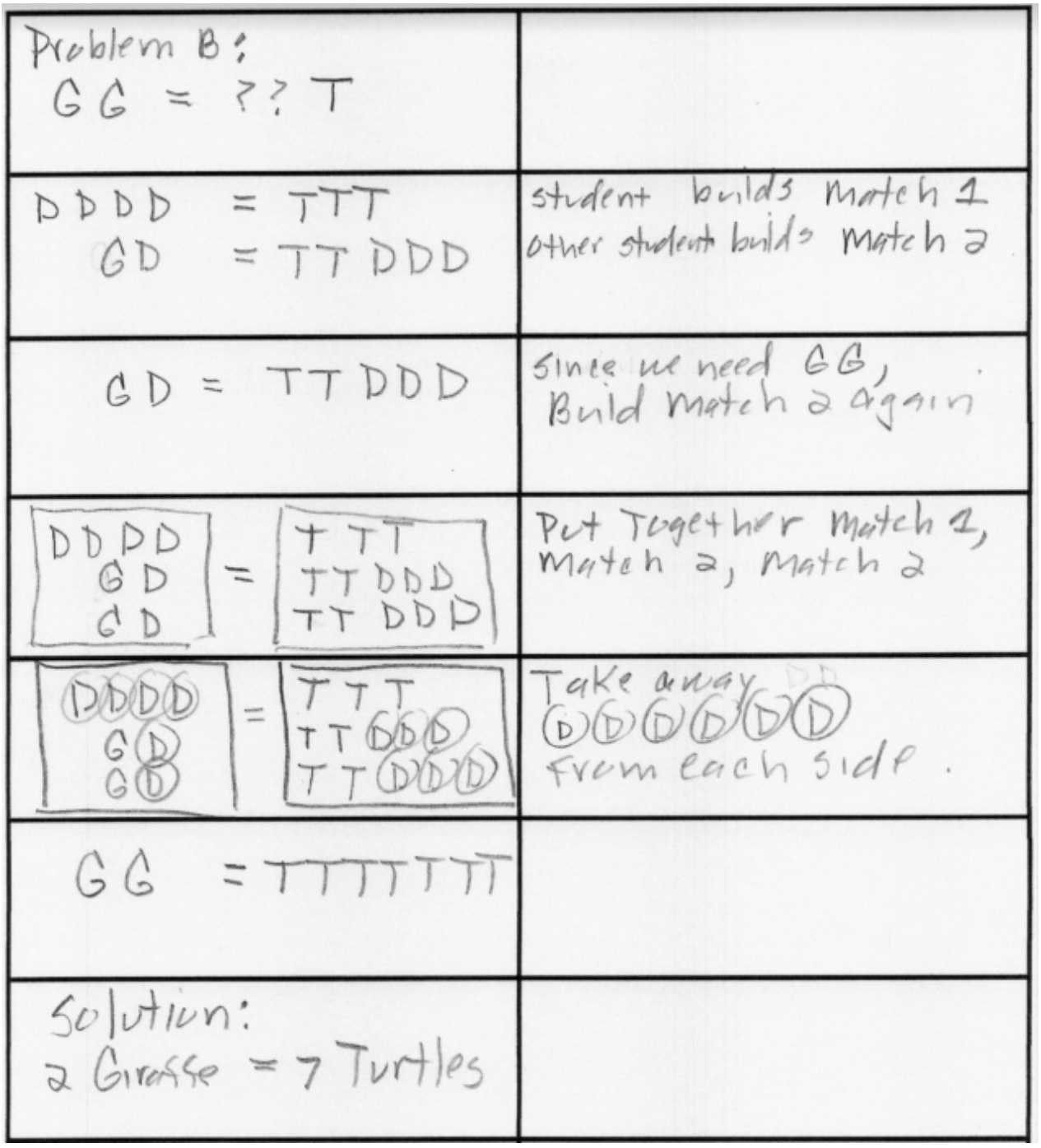
- “9 turtles” are equal in strength to “2 turtles and 2 giraffes”.

So if we remove 2 turtles from each team, we conclude:

- “**7 turtles**” are equal in strength to “2 giraffes”.

So the answer is: 7 turtles.

 Another solution:


Problem B:
GG = ?? T
DDDD = TTT
GD = TT DDD
student builds match 1
other student builds match 2
GD = TT DDD
since we need GG,
Build match 2 again
DDDD
GD
GD
=
T TT
TT DDD
TT DDD
Put Together match 1,
match 2, match 2
Take away
from each side.
GG = TTTTTTT
Solution:
2 Giraffe = 7 Turtles

C) Solutions

MATCH 5
Who wins?

MATCH 6
Who wins?

Match 5: Since 7 turtles are equivalent in strength to 2 giraffes, then each giraffe is equivalent to 3.5 turtles. Team E wins.

Match 6: From match 5, we know that one giraffe has the strength of 3.5 turtles.
From match 1, one duck has the strength of 3/4 (= 0.75) turtles.

So “1 giraffe and 2 ducks” have a strength of 4.25 giraffes. Team F wins.

ACTIVITY 2: FÚTBOL PRACTICE

Time: 25 minutes

2 Fútbol Practice

The starting eleven players of the *El Tri* (Mexican national Soccer team) wear jerseys with numbers from 1 to 11. There is a substitute player wearing 0.

We want to split the *El Tri* into two teams of 6 players each to play a practice game. We do so in a way that **the sum of the jersey numbers of the first team is equal to the sum of the jersey numbers of the second team**.

A *Dividing the El Tri in two teams*

Separate the *El Tri* players in two groups of 6 players each, so that the sum of jersey numbers for the two groups is the same.

How can this help you find the sum

$$T = 0+1+2+3+4+5+6+7+8+9+10+11$$

without having to add all numbers? Explain, and find T.

Value of T:

B *Players 0, 10 and 11 together*

In how many ways can we split the *El Tri* into two groups of 6 as in part A, so that the players 0, 10 and 11 (which are close friends) end up on the same team?

Dividing the Squad in three teams

C Separate the *El Tri* into three teams of 4 players each so that the sum of the jersey numbers is the same for each team. Which strategy did you use?

My strategy:

Use this to help you find (again) the sum T = 0+1+2+3+4+5+6+7+8+9+10+11 without adding all numbers. Verify that you got the same answer as before.

Value of T:

Team differences

D We have split the 12 players (0-11) in two teams of 6 each. For each such split, indicate which team has a larger sum of jerseys numbers and by how much (or if the two sums are equal) *without counting*. You are not allowed to compute the two sums (only do so at the end, to verify). The first was done as an example.

Team A	**Team B**	Larger?	By	Reasoning
0 3 4 7 8 11	1 2 5 6 9 10	They are EQUAL	0	We can match the values to even out differences of 1: 0<->1, 3<->2, 4<->5, 7<->6, 8<->9, 11<->10
0 1 9 7 12 6	11 3 2 10 5 4			
12 4 7 6 8 0	5 10 2 11 9 3			
Odd numbers	Even numbers			
#s less than 6	#s Larger than 5			

ACTIVITY 2: FÚTBOL PRACTICE	
Description	In this task, students will explore properties related to sums of the first 11 non-negative integers and discover patterns to balance them. It is not required that they know the formula for adding the first n positive integers, as this is not the main learning goal. Rather, the idea is to develop number sense, including reasoning about symmetries. However, some students may discover this formula at different points of the activity.
Materials	• Tokens numbered 0 to 11.
Set-up	Have students work individually or in groups of 2.
My solution	In this space, write your solution to the problems (working out details, not just the final answers). Use as many visual representations as possible! Also, write discussion questions: these are questions that help students, at the end, consolidate the math learning. **My solution** 

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 **My discussion questions (some examples are included)**

- From what you learned, what are some good ways to find the sum of the first integers up to a certain integer, without having to add them all? Illustrate with some concrete examples.
- ____
- ____

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<tr>
<td>Productive discussion</td>
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This section gives you examples of prompts, cues and questions that you may ask students during or at the end of the problem solving process.

- **If some groups are not able to "start" (overwhelmed)**
  - *"Tell me the problem in your own words, and draw a diagram to help you explain".*
    - This can help you find out to what extent the students comprehend the situation, and go from there.
- **If you see two students who seem shy or are working in isolation**
  - *"Hey Alan and Bianca, I see that you are working alone, maybe you want to work together for a while? I think you can learn a lot from each other"*
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Teaching tips	● You may first consider the same problem but only with players 0,1,2,3,4,5,6,7. This can make the computations much easier, whereas still revealing some patterns that are important in the original problem. ○ Likewise, you may extend the problem to, say, numbers 0 to 15, for those students that you think will be able to handle it. ● Encourage students to produce more than one solution if they can. The mindset of "finding more than one solution" can greatly help students to detect patterns, either by comparing similarities and differences in the solutions, and also because by finding solutions that are not that evident, one is usually forced into understanding the problem at another level. You may ask: ○ Can you make a small change to your solution to get a new one? Why is this also a solution? ● For part B, you may challenge students to find a splitting in two teams such that any three values are together (in the same way 0, 10 and 11 were together). Students can explore if that is always the case, or if they can find a counterexample. This can increase curiosity in students for finding general rules. ○ Example: for 0,1,2 together, you would do: 0,1,2,9,10,11. It works! ○ Example: for 4,5,6 you may do: 4,5,6,10,7,1. It works!

Solutions (2 Fútbol Practice)

A) One possible Splitting: We split 0 and 1 (so far the right team has 1 more). Then we add 3 to the left and 2 to the right (so we tie). We continue this way:

0 3 4 7 8 11	1 2 5 6 9 10

 Another solution: we first find the sum 0+1+2+3+4+5+6+7+8+9+10+11. It is easy to see that this is equal to (1+9) + (2 + 8) + (3 + 7) + (4 + 6) + 10 + 5 + 11 = 40 + 10 + 5 + 11 = 66.

Since we need the two teams to add to the same, they must add to 33 (half of 66) each. So we have reduced our task to find 6 numbers from 0 to 11 that add 33. For example:

9 + 1 + 8 + 2 + 7 + 6. So out solution is:

9 1 8 2 7 6	3 4 5 10 11

B) We need 0, 10 and 11 in the same team. One solution is to add three numbers (only one odd, because in 0, 10 and 11 there is only 1 odd) that also add to 21 in the other side, such as 8, 6 and 7:

0 10 11 . . .	8 6 7 . . .

Then we have 1, 2, 3, 4, 5, 9 left, which we need to split in two groups that add the same. For example: 1, 2, 9 and 3, 4, 5. The we complete the solution:

0 10 11 1 2 9	8 6 7 3 4 5

Or also:

0 10 11 3 4 5	8 6 7 1 2 9

 Another solution: Since we know that each sum of the group of 6 must be equal to 33 and 0+10+11=21, we need to find three numbers from 1 to 9 that add to 12 (as 21+12=33). This is easy. For example: 2, 4, 6. So this generates the solution:

0 10 11 2 4 6	1 3 5 7 8 9

C) *Separate the El Tri into three teams of 4 players each so that the sum of the jersey numbers is the same for each team.*

Solution: We balance as much as we can. We start with 0, 1, 2:

0 ...	1 ...	2 ...

Now we place 3, 4 and 5 to balance everything:

0 5 ...	1 5 ...	2 3 ...

We continue like this:

0 5 6 ...	1 5 7 ...	2 3 8 ...

Until we reach our splitting!

0 5 6 11	1 5 7 10	2 3 8 9

Of course, there are other solutions using this type of "balancing" strategy.

D)

Team A	**Team B**	**Larger?**	**By**	**Reasoning**
0 3 4 7 8 11	1 2 5 6 9 10	EQUAL	0	Cancel out +1 differences
0 1 9 7 12 6	11 3 2 10 5 4	EQUAL	0	Look for a +2 difference: {0 2} {1 3} {9 11} {7 5} { 12 10 } {6 4} These differences cancel out.
12 4 7 6 8 0	5 10 2 11 9 3	B	3	+2 differences (for A): {12 10 } { 4 2 } { 7 5 } +3 differences (for B) {6 9} {8 11} {0 3} Since 1+1+1=3, team B wins by 3.
Odd numbers	Even numbers	B	6	Six +1 differences are generated (for B): 0,1 2,3 4,5 etc. So B wins by 6
#s less than 6	#s larger than 5	B	36	Six +6 differences are generated (for B): 0,6 1,7 2,8 etc. So B wins by 36.