5

TEACHING MANUAL

MEET & MATH 5

FALL 2018 MEETING 5

OCTOBER 30-31

Contents

www.math.uci.edu/mathceo

2018 UCI MATH CEO COMMUNITY EDUCATIONAL OUTREACH.
UNIVERSITY OF CALIFORNIA AT IRVINE

- Tuesday 9:00 AM - 9:50 AM: October 30 (UCI Week 3)
    - Place: **UCI** NS 2 1201 (Marco Forster comes)
- Tuesday 2:45 PM - 3:45 PM: October 30 (UCI Week 3)
    - Place: **SANTA ANA**: Carr Intermediate School
- Wednesday 2:00 PM - 3:45 PM: October 31 (UCI Week 3):
    - Place 1: **UCI,** NS2 1201 (Lathrop comes)
    - Place 2: **UCI,** ALP 2600 : new Anteater Learning Pavillon building (Villa comes)

Tuesday 10/30 , 9AM (50+ minutes) ● **Activity 1**: 40 minutes ● **Survey**: 5 minutes ○ Start at 9:40 AM (or 3:35 PM) *Skip Activity 2* Note: David Wych will be giving an ongoing CRASH course from 8:45 - 9:00 on Tuesdays (just before the 9:00 AM meeting at NS2 1201)	Wednesday 10/31 (80+ minutes) ● **Activity 1**: 45 minutes ● **Activity 2**: 15 minutes ○ Only start this activity if time is 3:15 or earlier ● **Survey**: 5 minutes ○ Start at 3:40 PM
Tuesday 10/23 , 2:45 PM (50+ minutes) ● **Activity 1**: 40 minutes ● **Survey**: 5 minutes ○ Start at 9:40 AM (or 3:35 PM) *Skip Activity 2*	

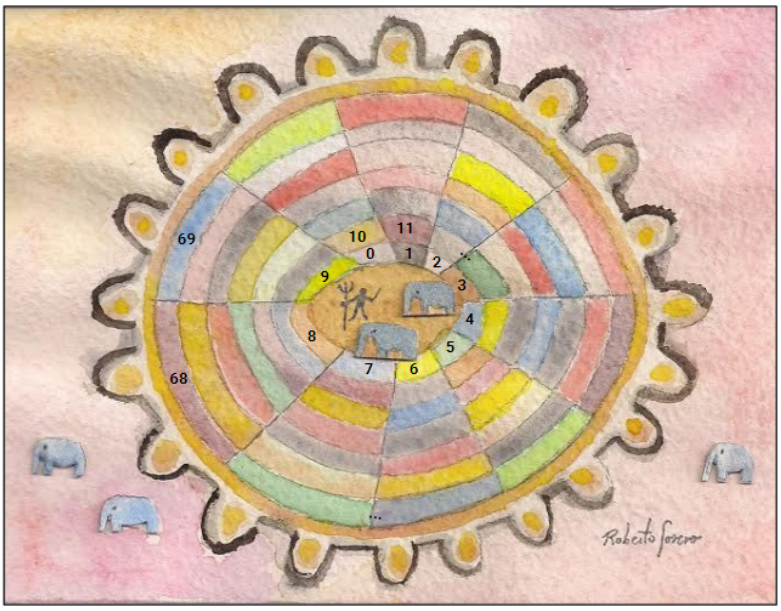
Meeting 5

Activity 1: The Colosseum
Time: 45 minutes
https://www.math.uci.edu/mathceo/Files/Other/Meeting1_2015-2016.pdf

Activity 2: Memory Game
Time: 30 minutes
http://map.mathshell.org/tasks.php?unit=MA20&collection=9

2 The Colosseum

Marco, Alessia and Bruno start their tour of Italy in the beautiful Rome, the capital and heart of Italy, heading to the Colosseum in a warm sunny day..



The Colosseum has a **total of 70 zones**, numbered from 0 to 69 as the picture shows. A group of doves and crows fly to the Colosseum, landing on some of these zones.

Marco notices the following:

- there is a dove every 5 zones, starting from the second one (zones 2, 7, 12 and so on).
- There is a crow every 4 zones, starting from the second one (zones 2, 6, 10 and so on).

Doves and Crows

a How many zones have a dove?

b How many zones have a crow?

Write your process and answers in your notebook

Continues...

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69

c How many doves are in the interval between zones 9 and 22 (including endpoints)? Find an interval as large as you can containing 5 doves.

d Which zones have both birds? List them all.

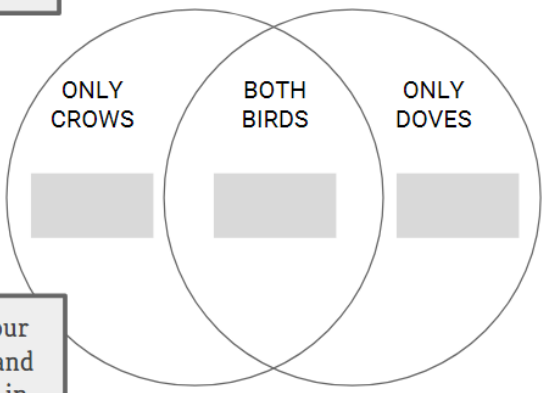
e How many zones have neither bird?

Write your process and answers in your notebook

A Larger Colosseum

CHALLENGE

Suppose that the Colosseum has 410 zones (from 0 to 409) instead of 70. Complete the following diagram, indicating the number of zones with only doves, only crows, both birds and no birds. Note that these numbers should add to 410.



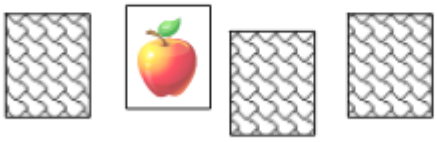
Write your process and answers in your notebook

Memory Game

Ella is teaching her little brother Sammy to play a memory game.

1. She starts with 4 cards, 2 have a picture of an apple on them and 2 have a picture of bananas.

   The cards are laid on the table with the pictures hidden, and then mixed up.
   Ella says 'you can turn over 2 cards and you win if they are the same'.

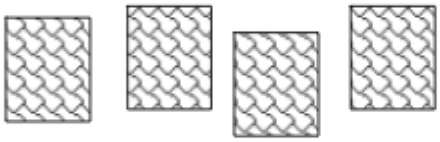


   Sammy turns over 1 card, it has an apple on it, then turns another.
   What is the probability that the second card has an apple on it? _____
   Explain how you figured it out.

_____

_____

2. Sammy did not get 2 cards the same so Ella turns the cards back over so that the pictures do not show, but she did not mix up the cards.



   Sammy is good at remembering where things are.
   He turns over 1 card and then another.
   How should he choose the first card to turn over so that he can be sure to win?

_____

_____

3. Ella adds 2 cards with pictures of an orange, making 6 cards all together.

She lays all the cards down with the pictures hidden and mixes them up.

Sammy turns over 1 card and then another.

What is the probability that they are both the same? _____
Explain how you figured it out.

_____

_____

4. What is the probability that they both have pictures of a banana? _____
Show how you worked it out.

ACTIVITY 1: THE COLOSSEUM	
Description	In this task, students will explore topics of divisibility, counting and number patterns. The context is a trip to Italy in which students notice patterns and need to come up with strategies for counting different elements. This counting can be done by "brute force" as first approaches, but students should be encouraged to see and use patterns, at some point. The main learning goal of this activity, which you can read to students, is: *We can identify patterns to develop multiple ways to be able to count things more efficiently.* Other goals include: ● Students can group a large set into parts with similar structure and use this to solve problems. ● Students can identify grid patterns and regularities related to periodicity and multiplication. ● Students develop number sense through real-world explorations
Materials	● Student's Workbook ● Large Colosseum (1 per table)
Set up	● Students can work in groups of 2-3 students. Encourage interactions between peers in terms of explaining plans on how to solve the different problems in the activity, which are increasingly more difficult.
My solution	In this space, write your solution to the problem (working out details, not just the final answers). Use as many different approaches as possible! Also, write **discussion questions**: these are questions that help students, at the end, consolidate the math learning. **My solution** 

My discussion questions (some examples are included)

-

Write your own discussion questions here:

- ____
- ____

Productive discussion	This section gives you examples of prompts, cues and questions that you may ask students during or at the end of the problem solving process. Before you continue, please watch:

[Communication in the Teaching and Learning of Math]
More Math 192 Series Videos:
(www.math.uci.edu/mathceo/teachingvideos.php)

- **If some students are stuck and cannot begin to make progress**
  - Make sure that students know the meaning of the word “Zone”.
  - You can have students draw the different birds with different colors. Or even have them point at them with their fingers.

- **Providing scaffolding**
  - Ask students: “How many zones are there from 0 to 9?” (it’s important that they recognize that there ar 10 and not 9 zones in the interval [0,9].
  - Then you can give students cues to indicate that thinking about zones of 10 is a good problem-solving strategy (or even thinking about zones of 20, like [0,19], etc.

Teaching tips

- A general question to get your students started is, “How can I organize the information given?” You can then go through the paragraph in chunks. For example:
  - “So Marco notices that there is a dove **every 5 zones starting from the second zone.** How can I organize this?”
  - Change your tone of voice to emphasize key information.
- Point out the resources in their notebook and give them suggestions on how to use it.
  - “Your notebook has the numbers 0-69 written out, how can you use that to help you organize your thoughts?”
- Remember that the answer is not as important as their understanding. Instead of giving quick fixes, try to always ask them for their reasoning.
  - Ask more “why” questions than “what questions.”

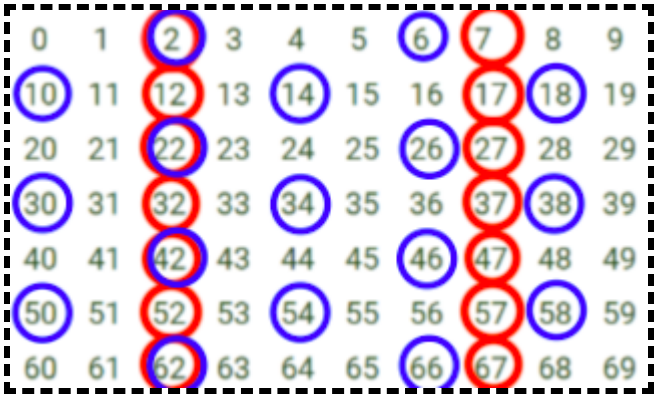
Going deeper (optional)

Home challenge:
In this challenge, the kids will learn the cultural similarities between the Hispanic and Italian culture. It will also encourage them to travel as part of their life-long learning.

Solutions (The Colosseum)

See also: 

Colosseum Zones



The zones with a **dove** are in **Red**; the zones with a **crow** are in **Blue**.

We can see clearly, from marking the zones by bird, that there are **2 doves** for every **10 zones**.

The **crows** are a bit trickier: there are **2 crows** in some 10-zone rows, and **3 crows** in some 10-zone rows. If forget about looking row-by-row, and look instead at **2 rows at a time**, we can see that there are **5 crows** for every 2 10-zone rows (**20 zones**).
Or, alternatively, we see that rows 1, 3, 5, and 7 have **2 crows**, and rows 2, 4, and 6 have **3 crows**.

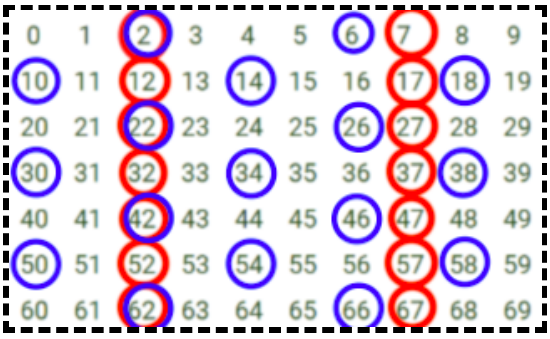
a) Colosseum has zones 0 to 69 (**7 rows, 10 zones / row = 70 zones**):
2 doves / 10 zones → (2*7) doves / (10*7) zones = 14 doves / 70 zones

 Explanation in words: The doves land in the zones 2, 7, 12, 17, 22, 27, … Notice the pattern: there are exactly 2 doves for every block of ten zones (1→10, 11→20, 21→30, etc), for a total of 14 doves.

b) Colosseum has zones 0 to 69 (**7 rows, 10 zones / row = 70 zones**):
- Rows 1, 3, 5, and 7 (4 rows, 10 zones / row) → 2 **crows** / 10 zones → (2*4) **crows** / (10 * 4) zones
  = 8 crows / 40 zones
- Rows 2, 4, and 6 (3 rows, 10 zones / row) → 3 **crows** / 10 zones → (3*3) **crows** / (10*3) zones
  = 9 crows / 30 zones

- **Putting it all together:**
  (8 + 9) **crows** / (40 + 30) zones = **17 crows / 70 zones**

Another way: The crows, land in the zones 2, 6, 10, 14, 18, 22, 26, 30, 24, 28, 32, 36, 40, 44, 48, …. In order to see the pattern for the crows, we need to consider blocks of 20 zones (1→20, 21→40, 41→60). Every such block has 5 crows; hence there are 15 crows in zones 1→ 60, plus two more between 61 and 69, for a total of 18.



c) **An approximation:**
There are two ways to handle the number of **doves** in **zones 9 to 22**:

1. Simply look at our colosseum chart and count them: there are **3 doves** in this interval
2. How many zones? 22-9 = **13 zones (+1 for the end-point) = 14 zones**
   There are **2 doves** for every **10 zones** → 2 doves / 10 zones =
   (2*1.4) doves / (10*1.4) zones = 2.8 doves / 14 zones → round up →
   3 doves / 14 zones
   - **Why doesn't this work *exactly*? What makes this an *approximation* and not an *exact* answer? What does the 10-row frequency *miss*?**

Finding the largest interval with 5 doves:

- Starting at a zone with a **dove**: start at **zone 7**: **5 doves** → zones 7, 12, 17, 22, 27 → 27-7 = 20 zones (+1 for the end point) = **21 zones**
- Starting in a zone without a **dove**: start at **zone 8** → zones 12, 17, 22, 27, 32 → 32-8 = 24 (+1 for the end point) = **25 zones**
- Starting at *another* zone without a **dove**: start at **zone 10** → zones 12, 17, 22, 27, 32 → 32-10 = 22 zones (+1 for the end points) = **23 zones**
  - **Now we are going down in the number of zones? Where does this pattern come from? What if we started from zone 9, or 11? How many zones would we have to count to get 5 doves?**

Another way: visual: The largest number of consecutive zones containing exactly 5 doves is 28:

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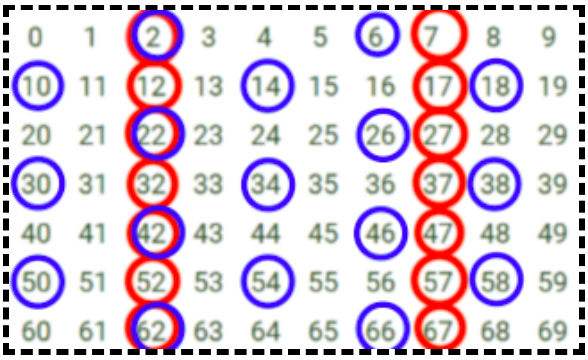
d) **Which zones have both doves and crows?**



- If we look at our colosseum chart: zones 2, 22, 42, 62. **What's the pattern?** Well first, it's only the even number rows, but then there is also a number-based pattern:
  Pattern → (first zone of even numbered row) + 2:
  - Row starting with 0 → 0 + 2 = **2**
  - Row starting with 20 → 20 + 2 = **22**
  - Row starting with 40 → 40 + 2 = **42**
  - …

 Another way to see it: The zones with the doves have numbers of the form "2 + a multiple of 5", the ones with the crows have numbers of the form "2 + a multiple of 4". Since the *lowest common multiple* of 4 and 5 is 20, the zones containing both birds have numbers of the form "2 plus a multiple of 20": 2, 22, 42, 62.

e) How many zones have neither bird? Notice the pattern:

- "Odd" rows (1, 3, 5, 7): **7** zones have neither bird [example: Row 0-9]
  - This gives 4*7 = **28** zones having no birds
- "Even" rows (2, 4, 6): **5** zones have neither bird [example: Row 10-19]
  - This gives 3*5 = **15** zones have no birds.
- Adding: 28 + 15 = **43** zones have neither (we can add, because the zones above do not share any common parts: the rows do not "intersect").

Here is a table that shows the pattern described above:

Row	1sts	2nd	3rd	4th	5th	6th	7th	
Zones	0-9	10-19	20-29	30-39	40-49	50-59	60-69	TOTAL
# of zones with no birds	7	5	7	5	7	5	7	(7+5)x3 + 7 = 36+ 7 = **43**

 Another way to see it: blocks of twenty...
Every block of 20 zones (0→19, 20→39, 40→59) contains 12 zones with no bird:
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20.
Of the remaining 10 zones (60→69), seven have no bird: **60**, **61**, 62, **63**, **64**, **65**, 66, 67, **68**, **69**.
Hence there are a total of 36+7 = **43** zones with no birds.

-v-v-v- CHALLENGE -v-v-v-

Challenge: 410 zones (zones 0 to 409)

- How many rows?
  - 10 zones / row → 410/10 = **41 rows** (starting with 0, 10, 20, … 100, 110, … 390, 400)
    - 21 even numbered rows (starting with 0, 20, 40, … 380, 400)
      - **2 doves, 2 crows, 1** with **both, 7** with **neither**
      - 2*21 = **42 doves;** 2*21 = **42 crows**; 1*21 = **21 both**; 7*21 = **147 neither**
    - 20 odd numbered rows: (starting with 10, 30, 50 … 370, 390)
      - **2 doves, 3 crows, 0** with **both, 5** with **neither**
      - 2*20 = **40 doves**; 3*20 = **60 crows**; 0*20 = **0 both**; 5*20 = **100 neither**

Totals:

- 42 + 40 = **82 doves**
- 42 + 60 = **102 crows**
- 21 + 0 = **21 both**
- 147+100 = **247 neither**

ACTIVITY 2: MEMORY GAME	
Description	In this task, students find probabilities for different events in the context of drawing cards from a Memory Game.
Materials	• Student's Workbook • Deck of cards (1 per table)
Set up	• Students can work individually and check their answers with mentors or other students, whenever they want. But they should start working individually. • Make sure students use the cards. However, at first, do not tell them how exactly. Just let them know that they can use this resource to make sense of the problem or to explain their solutions.
My solution	In this space, write your solution to the problem (working out details, not just the final answers). Use as many different approaches as possible! Also, write **discussion questions**: these are questions that help students, at the end, consolidate the math learning. **My solution** 

My discussion questions (some examples are included)

- How would this problem change if we changed the number of each type of card? (Discuss at least one variation of the problem).

Write your own discussion questions here:

- ____
____
____

- ____
____
____

Productive discussion

This section gives you examples of prompts, cues and questions that you may ask students during or at the end of the problem solving process.

Before you continue, please watch:

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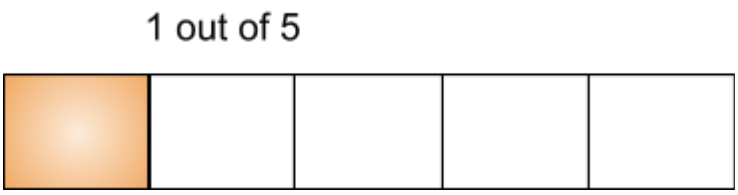
More Math 192 Series Videos:
(www.math.uci.edu/mathceo/teachingvideos.php)

- **If some students are stuck and cannot begin to make progress**
  - Some students may not remember what probability means or how can they find a probability as a fraction. It is ok to explain the idea to them by doing some examples. Include pictures in these examples. For example, you can illustrate how the probability of selecting an even number (at random) from the set {5,6,7,8} is one half, by drawing this set and making sense of 2/4.

	● **Providing scaffolding** ○ In this problem, the probabilities have denominators 3, 5 and 15. You can teach students how to make sense of this, by drawing a fraction bar with the probability. For example, for the 1/5 probability: 
Teaching tips	● Don't assume that students know what is the conceptual meaning of a probability, or that they know how to compute a probability. ○ Some key questions so that students make sense of a computed probability: ■ Is it more likely that this happens, or that it does not happen (or equally likely)? (This corresponds to asking if the probability is more than 1/2 or 50%)

Solutions (Memory game)

See also: http://map.mathshell.org/download.php?fileid=1119

Memory Game
1. Gives correct answer: **1/3** or 33.3% Gives a correct explanation such as: One of the remaining three cards has an apple on it.
2. Gives correct answer such as: One of the cards he did not turn over the first time.
3. Gives correct answer: **1/5** or **20%** Gives a correct explanation such as: It does not matter which card he turns over first, when one card has been turned over there are 5 left and only 1 of those will make the pair. May draw tree diagram.
4. Gives correct answer: **1/15** or 6.6% Shows correct work such as: 2/6 x 1/5. May list possibilities or draw tree diagram.