TEACHING MANUAL

MEET & MATH 3

FALL 2018 MEETING 3

OCTOBER 16-17

Contents

1) COUNTING TREES

2) CANDLE BOX

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2018 UCI MATH CEO COMMUNITY EDUCATIONAL OUTREACH.
UNIVERSITY OF CALIFORNIA AT IRVINE

Teaching Manual

Student workbook

Meeting 3 (10/17)

- Tuesday 9:00 AM - 9:50 AM: October 16 (UCI Week 3)
    - Place: **UCI** NS 2 1201 (Marco Forster comes)
- Tuesday 2:45 PM - 3:45 PM: October 16 (UCI Week 3)
    - Place: **SANTA ANA**: [Carr Intermediate School](#)
- Wednesday 2:00 PM - 3:45 PM: October 17 (UCI Week 3):
    - Place 1: **UCI,** NS2 1201 (Lathrop comes)
    - Place 2: **UCI,** ALP 2600 : new Anteater Learning Pavillon building (Villa comes)

Tuesday 10/16 (50+ minutes)	Wednesday 10/17 (80+ minutes)
Activity 1: 40 minutes **Survey**: 5 minutes - Start at 9:40 AM (or 3:35 PM) *Skip Activity 2* Note: David Wych will be giving an ongoing CRASH course from 8:45 - 9:00 on Tuesdays (just before the 9:00 AM meeting at NS2 1201)	**Activity 1**: 45 minutes **Activity 2**: 15 minutes - Only start this activity if time is 3:00 or earlier **Survey**: 5 minutes - Start at 3:35 PM

Activity 1: Counting Trees

Time: 45 minutes
http://map.mathshell.org/download.php?fileid=1654

Activity 2: Candle Box

Time: 30 minutes
http://map.mathshell.org/tasks.php?unit=ME01&collection=9

Counting Trees

The diagram shows some trees in a tree farm.

The circles ● show old trees and the triangles ▲ show young trees.

Tom wants to know how many trees there are of each type, but says it would take too long counting them all, one by one.

1. What method could Tom use to estimate the number of trees of each type?

   Explain your method fully.

   ....
   ....
   ....
   ....
   ....
   ....
   ....

2. Use your method to estimate the number of:

(a) Old trees.

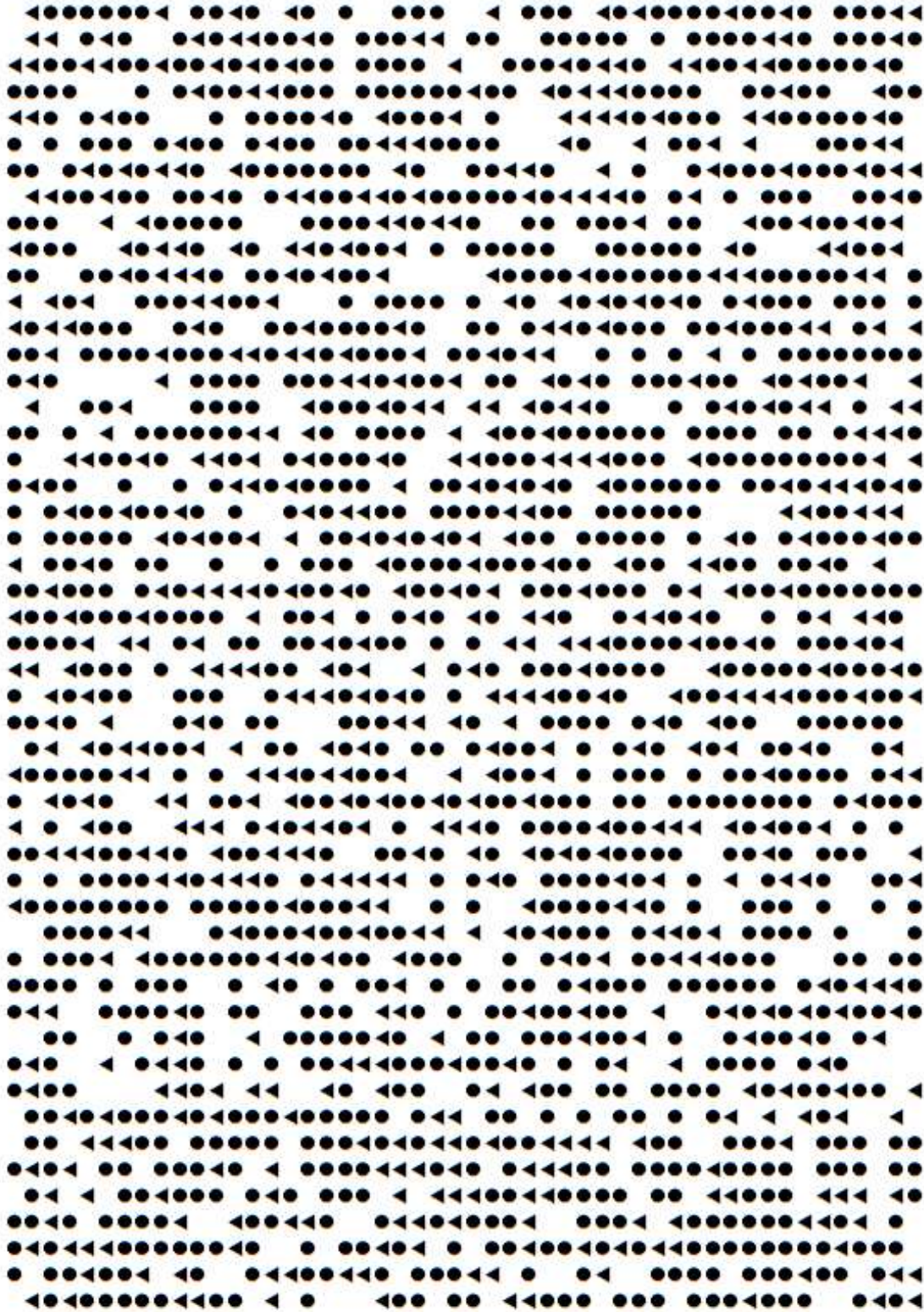   ....
   ....
   ....
   ....
   ....

(b) Young trees.

   ....
   ....
   ....
   ....
   ....

The Tree Farm

	COUNTING TREES
Description	In this task, students are shown a picture with lots of young and old trees (represented by icons) and they need to estimate how many of each are there. Goals: students should be able to: ● Solve simple problems involving ratio and direct proportion. ● Choose an appropriate sampling method. ● Collect discrete data and record them using a frequency table.
Materials	● Student's workbook ● Extra "Farms"

Set up	1) **Assessment task: 20 minutes**
	(10 minutes "solo" + 10 minutes "guided")
	    a) Briefly introduce the task: Does anyone know what a tree farm is? How is a tree farm different from a natural forest? [The trees are deliberately grown for commercial purposes; e.g. building or paper. They are often planted in rows.]
	    b) Give each student a copy of Counting Trees and The Tree Farm.
	    c) Tell students:
	        ● *Spend 20 minutes on your own, reading through the questions and trying to answer them as carefully as you can.*
	        ● *I have extra copies of the sheet, The Tree Farm, if you need another copy.*
	        ● *Don't worry if you can't do everything in the first 20 minutes. Your goal is to be able to answer these questions with confidence by the end. I will help you a bit during the last half of these 20 minutes.*
	
	During student's work, carefully look at their work, but keep interaction as minimal as possible. You will need to pay attention to how students are approaching the problem.
	
	Guided phase (10 minutes)
	
	Based on what you are seeing on the students work, have them continue working on it, but, as they work, guide them with questions using the sheet in the next page, where you are addressing common issues related to the task:

Common issues:	Suggested questions and prompts:
Chooses a method which does not involve any sampling For example: The student counts the trees. Or: The student multiplies the number of columns by the number of rows and then halves this answer.	• Read the question again. Have you done what is asked? • What assumptions have you made? Are your assumptions reasonable? • How could you improve your estimate?
Chooses a sampling method that is unrepresentative For example: The student counts the trees in the first row/column and multiples by the number of rows/columns. Or: The student multiplies the number of trees in the left column by the number of trees in the bottom row.	• How could you improve/check your estimate? • Is your sample size reasonable? How do you know? • Which rows/columns have you left out of your calculations?
Uses area and perimeter in calculations	• What does the area measure? • What does the perimeter measure?
Makes incorrect assumptions For example: The student does not account for gaps. Or: The student does not realize that there are an unequal number of trees of each kind.	• Is there a pattern to how the trees are distributed in the tree farm? Does your work assume there is a pattern? • What does your method assume? Is this a reasonable assumption?
Calculates the number of trees in an unrepresentative sample area of the tree farm	• Is your sample area representative of the whole tree farm? • How could you check the accuracy of your estimate?
Produces work that is difficult to follow	• Would someone unfamiliar with the task understand your work?
Chooses an appropriate sampling method	• Can you suggest a second, different sampling method? • If you miscount your sample by 1, how does that affect your overall estimate?
Completes the task	• Now have a go at this problem. How many people can stand on a full-size tennis court? State your assumptions and come up with a reasonable estimate.

2) Group work (20 minutes)

Organize the class into small groups of two or three students and hand out an enlarged copy of The Tree Farm sheet, poster paper, a glue stick, and makers.

Ask students to have another go at the task, but this time ask them to combine their ideas and use what they have learned from reviewing their individual solutions.

*→ You each have your own individual solution and have been thinking about how you might improve this.*
→ Share your method with your partner(s) and your ideas for improving your work.
→ Together in your group, agree on the best method for completing the problem and produce a poster, which shows a joint solution to the task that is better than your individual solutions.
→ State on your poster any assumptions you have made and give clear reasons for your choice of method.

3) Whole-class discussion: sharing different approaches (5 minutes, if there is time left)

Depending on the state of your student's solutions, consider whether you want to wrap things up in a discussion, of you would rather do closing comments within each small group.
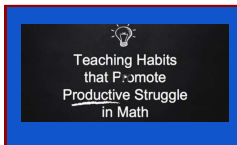
[Habits of productive struggle in math](#)
Productive struggle is the kind of effortful learning that develops grit and creative problem solving.

My solution	In this space, write your solution to the problem (working out details, not just the final answers). Use as many different approaches as possible! Also, write discussion questions: these are questions that help students, at the end, consolidate the math learning.

My solution

My discussion questions (some examples are included)

- Which new method did you learn from your mentor or from other students? Why did you enjoy discovering it?
- This task was not about finding an exact answer. In which ways, then, do you think that we were doing math?

Write your own discussion questions here:

- _____
  _____
  _____

- _____
  _____
  _____

Productive discussion	This section gives you examples of prompts, cues and questions that you may ask students during or at the end of the problem solving process.
	
	Before you continue, please watch:
	
	 Communication in the Teaching and Learning of Math
	More Math 192 Series Videos:
	(www.math.uci.edu/mathceo/teachingvideos.php)
	
	● **If some students are stuck and cannot begin to make progress**
	    ○ This is an open-ended task, so it is important to understand that there are several ways to solve it. During the first 10 minutes, kids should work individually and without guidance. However, if you feel that a student is not trying any method at all, then you can jump in with some questions, or just making sure that they understand the task. This being said, be very careful not to narrow down the exploration possibilities.
	        ■ We, as teachers, have sometimes a natural inclination to "tell the answer". This is specially true if you have, for example, tutored before. However, unfortunately, many times this **does not cause** learning, even though we feel so. We cannot do the thinking for the student.
	
	● **Providing scaffolding**
	
	    ○ After the first 10 minutes of students work, you may provide scaffolding by following the Suggested questions and prompts in the chart (see Setup section).
	    ○ Try not to make suggestions that move students towards a particular approach to the task. Instead, ask questions that help students to clarify their thinking.
	        ■ What assumptions have you made?
	        ■ Do you think they are reasonable?
	        ■ Why did you select this sample to count?
	        ■ Why is it helpful to count trees in more than one sample?
	        ■ How do you decide how many sample areas to count? [If there is a big variation in the number of trees then students may want to count the number of trees in a third area.]
	        ■ What is the difference between an estimate and a guess?How can you check you have a good estimate?

Teaching tips	● It is important that students are allowed to answer the questions without assistance, as far as possible.
	    ○ If students are struggling to get started then ask questions that help them understand what is required, but make sure you do not do the task for them.
	    ○ You may need to remind students that Tom does not want to count all the trees in the tree farm.

	● Encourage students, at some point, to use color codes or other ways to make sense of the information.
	
	● Students who sit together often produce similar answers, and then when they come to compare their work, they have little to discuss.
	  ○ So we suggest that when students do the task individually, you ask them to move to different seats. Experience has shown that this produces more profitable discussions.
	
	● While students work in small group, note their different approaches to the task and support student reasoning.
	  ○ In particular, note whether students' original methods are the same or different to their joint strategy.
	    ■ How do they decide which method to use?
	    ■ What assumptions do they make?
	    ■ Do students choose an appropriate sampling method?
	    ■ Do they check their estimate by considering a different sample?
	    ■ What makes them decide they have looked at enough sample areas?

Reflection tasks	Mentor reflection: Complete the following **Sample Responses to Discuss**: See next pages. (Complete the questions.)

Sample Responses to Discuss: Laura

① You could multiply the number of trees in the length by the number of trees in the width and then ½ half your answer.

② a. Old trees − 644
   Young trees − 644

   width − 33   33×39 = 1287
   length − 39.   1287 ÷ 2 = 643.5 − 644

Does Laura's approach make mathematical sense? Why does she halve her answer?

What assumptions has Laura made?

In what ways could Laura's work be improved?

To help you to understand Laura's work, what question(s) could you ask her?

Sampling and Estimating: *Counting Trees*
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Sample Responses to Discuss: Wayne

2 columns has 21 young trees
                  55 old

50 columns is approx
50 ÷ 2 = 25
25 × 21 = amount of young trees = 525
25 × 55 = amount of old trees = 1,375
          rounded up
young 530
old   1,380

Does Wayne's approach make mathematical sense? Why has he multiplied by 25?

What assumptions has Wayne made?

In what ways could Wayne's work be improved?

To help you to understand Wayne's work, what question(s) could you ask him?

Sample Responses to Discuss: Amber

Counting trees

1. If Tom draws a 10X10 square round some trees and counts how many old and new there are. There are 50 rows and 50 columns altogether so he must multiply by 25. He could do this a few times to check and then take the average.

2.
53 old	x 25	=	1325 old
28 new	x 25	=	700 new
19 spaces	x 25	=	475 spaces
100			2500

1325 + 1200 ÷ 2 = 1262·5
700 + 875 ÷ 2 = 787·5

check

48 old	x	25	=	1200 old
35 new	x	25	=	875 new
17 spaces	x	25	=	425 spaces
100				2500

So about 1263 old trees and 788 new trees

Does Amber's approach make mathematical sense? Why has she multiplied by 25?

In what ways could Amber's work be improved?

To help you to understand Amber's work, what question(s) could you ask her?

Student materials   Sampling and Estimating: *Counting Trees*   S-5
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Going deeper (optional)

Now have a go at this problem. How many people can stand comfortably on a full-size tennis court? The court is 78 feet (23.77 metres) long. Its width is 27 feet.
State your assumptions and come up with a reasonable estimate.

Solutions (Counting trees)

See also: http://map.mathshell.org/download.php?fileid=1654

There are many ways of completing this task, but solutions should include the following:

- Students should describe an appropriate sampling technique which takes into account the different proportions of old and young trees as well as allowing for the gaps in between.
- Students should check their figures for the number of old and young trees by counting the number of trees in at least two sample areas. •If there is a big variation in the number of trees in two areas then students should count the number of trees a third or subsequent sample area.
- Each sample should cover a distinctly different area of the tree farm (that is two sample areas should not be next to each other.)
- Each sample should cover the same sized area.
- Students may calculate the average number of old and young trees for their chosen samples. These averages should then be multiplied up to obtain an estimate for the total number of old and young trees.

Before you start: For any method, it will be important to know the total number of slots. This can be accomplished by counting the number of rows and columns, and multiplying the two:
- **Columns** = 50, **Rows** = 50 → 50 x 50 = **2500 total slots**

Also, important: **why do we need to estimate?**

- **DO THIS WITH YOUR STUDENTS IF YOU HAVE TIME:** it's an important skill to be able to figure out when/if a method you might use to solve a problem will be intractable **ahead of time.**

- There are 2500 slots. Even if we count 2 slots a second and record the number of old and young trees as we go, it will take 2500/2 = 1250 seconds = 1250/60 = ~20 minutes to count the young and old trees in each farm. **There must be a better way.**

v v v   Methods below   v v v

🐵 Method 1: Rows/Columns

- Have the students close their eyes and randomly plop their pen/pencil down on the paper to randomly select a row or column, and count the young/old trees in that row/column. Have them do this 5 times. Landing outside the page selects the outermost row/column.
    - **Question:** Why do this randomly instead of just choosing the first 5 rows/columns? Is there any (potential) differences between these two methods?
- Multiply the number of young trees and old trees they find in the 5 rows or columns by 10
    - **Question:** Why 10?
- The number of young and old trees **should not** add up to 2500 (why?).
    - **Question:** What accounts for the difference? Does the number of non-tree slots match with your estimation of how many empty slots there were in the randomly-chosen rows/columns?

Example: Columns

The Tree Farm
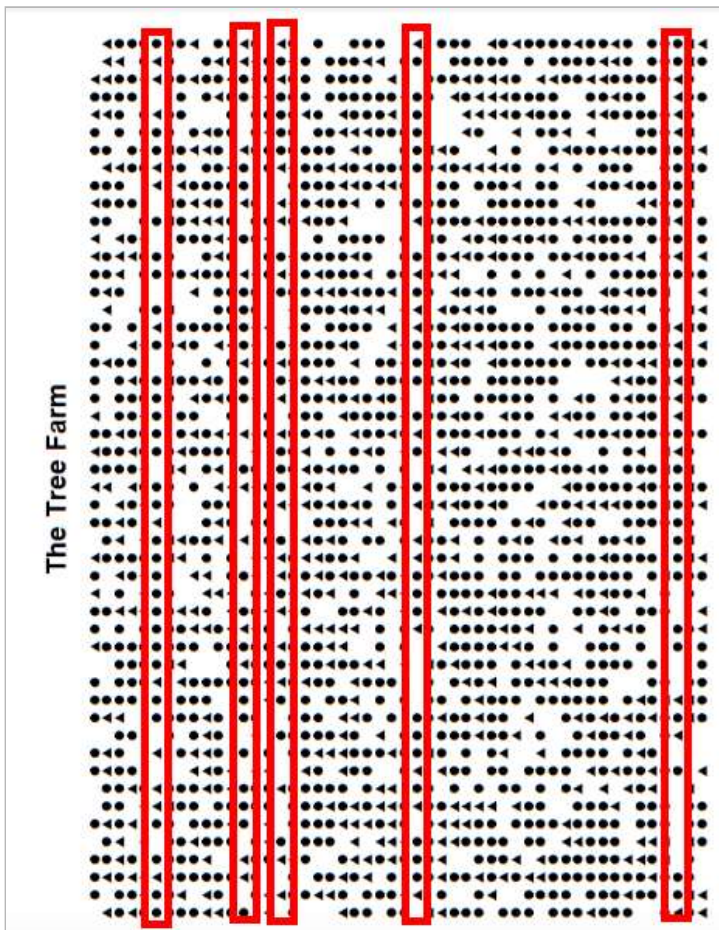
First column: 29 old trees, 12 young trees (9 empty)
Second column: 23 old trees, 12 young trees (15 empty)
Third column: 18 old trees, 17 young trees (15 empty)
Fourth column: 25 old trees, 13 young trees (12 empty)
Fifth column: 23 old trees, 15 young trees (12 empty)

Totals in sample: 118 old trees, 69 young trees
Multiply by 10: 1180 old trees, 690 young trees -- total of **1870** trees (630 empty slots)
    - **Question:** Does the number of empty slots match up with the estimate of empty slots per row/column?

🐵 Method 2: 5x5 squares

- Have the students close their eyes and plop down their pencils/pens randomly on to the page. Have them draw a **5x5 square of slots** around the place they plopped the pen/pencil down. Have the students do this **two times**. Have the students count the number of young and old trees in these two squares. (If they land off the page, have them select a 5x5 square closest to where the pen/pencil plopped down)
    - **Question:** Is it okay if there is overlap between the two squares? Why or why not?
- Have the students multiply the number of young/old trees in the two squares they selected by **50** (why 50?)
    - **Question:** What are the differences between doing the row/column method and this 5x5 square method? Is one method better/worse than the other? Why or why not?

Example:
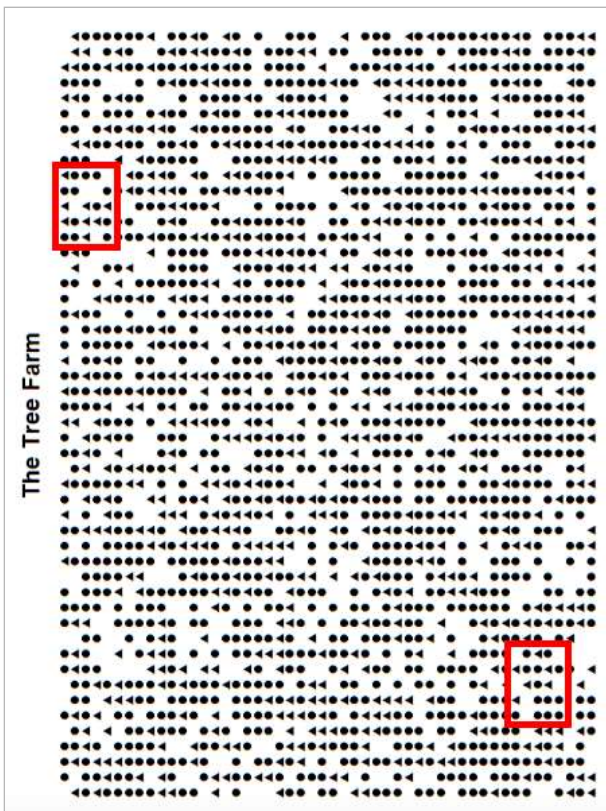
- **Top Left Square:** 12 old trees, 8 young trees (5 empty)
- **Bottom Right Square:** 13 old trees, 6 young trees (6 empty slots)

- **Totals in sample**: 25 old trees, 14 young trees
- **Multiply by 50:** 1,250 old trees, 700 young trees = **1,950** total trees (550 empty slots)

Questions for both methods

- How do we know if the sample rows/columns/squares we chose are **representative** of the whole farm?
    - If you choose a row/column/square with more young trees/old trees than a previous row/column/square, **does that mean you chose badly**? Do you need to find a new row/column/square?
- Which method above is **faster**? Which is **more accurate**? How can we know? Can we know without counting out all the trees? (Estimation vs. real answers)
- How could we make each method better? (More samples? *Better samples*? How/why would they be better?)

CANDLE BOX
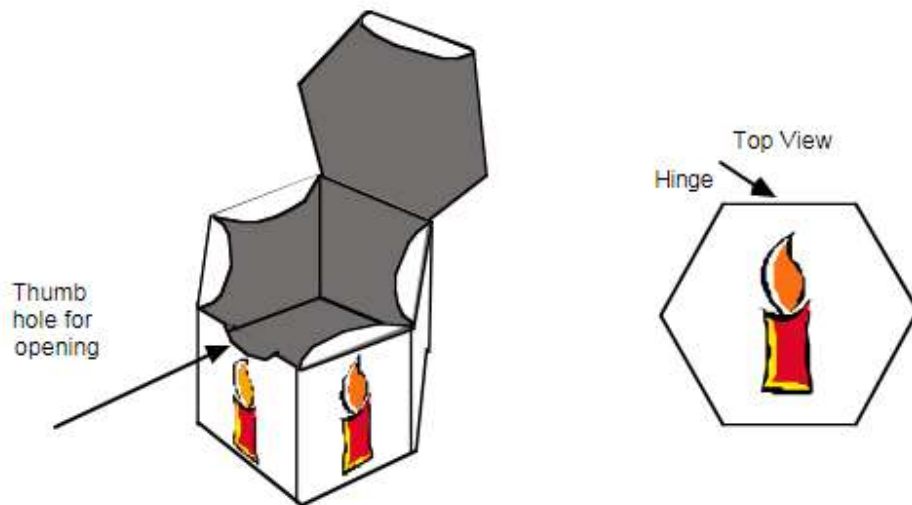
Tom is making a little gift box to hold a candle.

I want the top and the base to be regular hexagons.
The sides will be rectangles.
A little candle design will go on each side.
There will be a thumb hole to help you open the box.

Draw an accurate plan for making the box using the dotted paper provided.

It should be drawn so that when it is cut out it will stay in one piece.
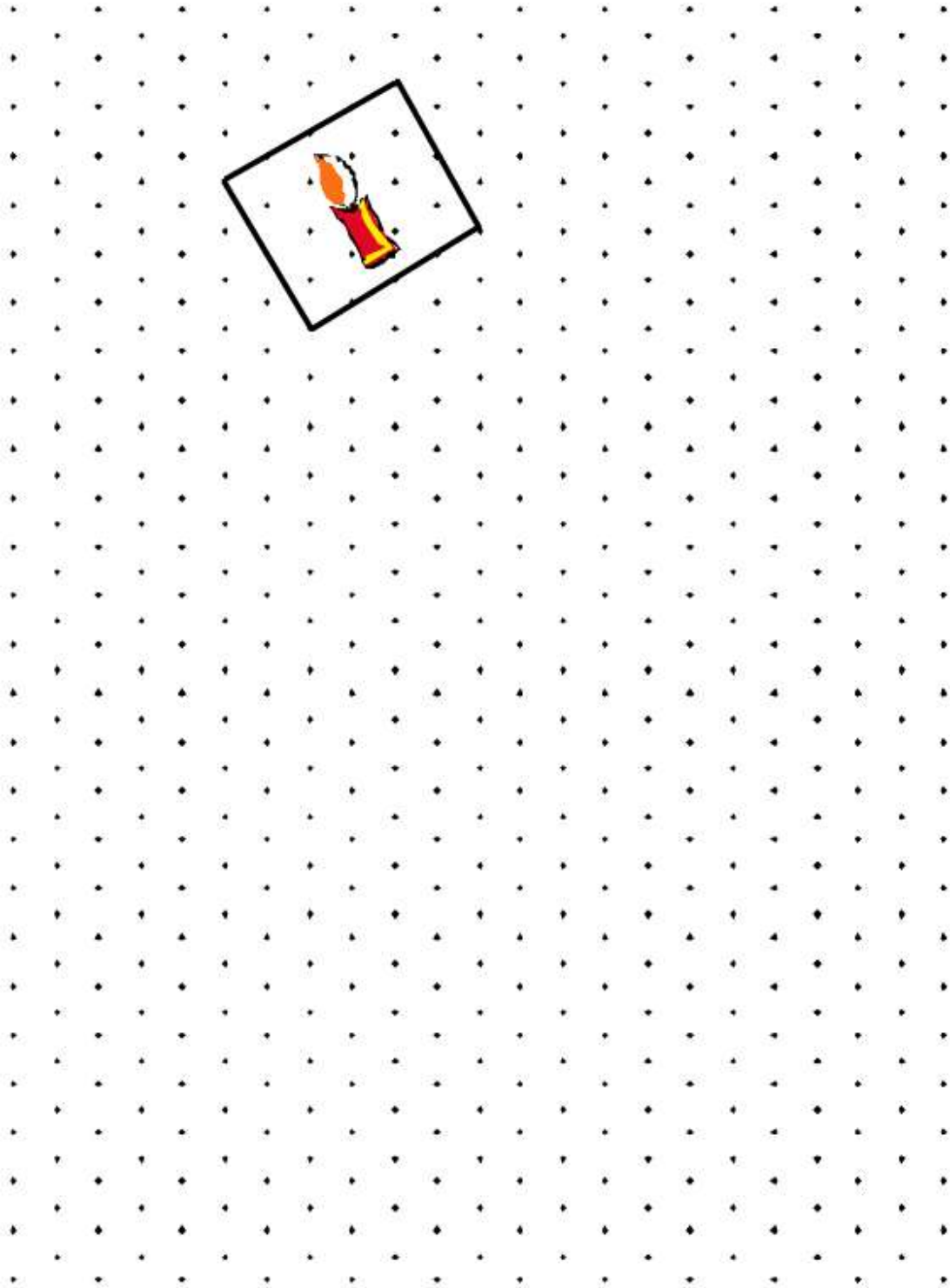
One of the sides has been drawn to start you off.

Make sure that you show:
- the flaps needed for gluing the box together. Shade these in.
- the flaps needed for fastening the lid (Do not shade these in),
- the thumb hole,
- the picture of the Birthday Candle the right way up on each side, and on the lid.

CANDLE BOX WORKSHEET

Draw your design below:

	CANDLE BOX
Description	In this task, students need to draw a net for a candle box, meeting certain conditions for their sides.
Materials	- Student's workbook - Rulers - Scissors
Set up	Students can work any way they want. For example, in pairs.
My solution	In this space, write your solution to the problem (working out details, not just the final answers). Any clarification or side notes that you consider helpful! Also, write discussion questions: these are questions that help students, at the end, consolidate the math learning. **My solution**
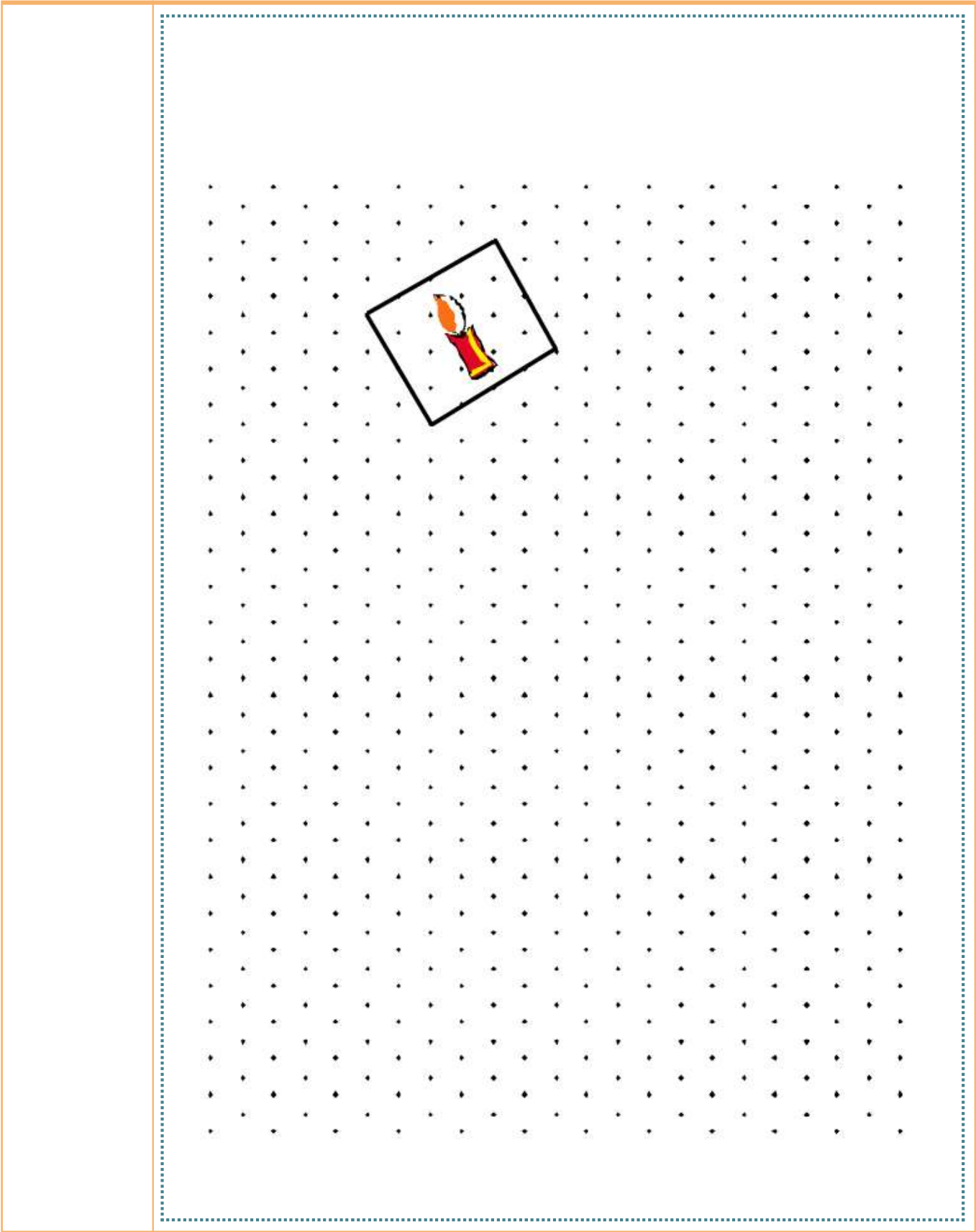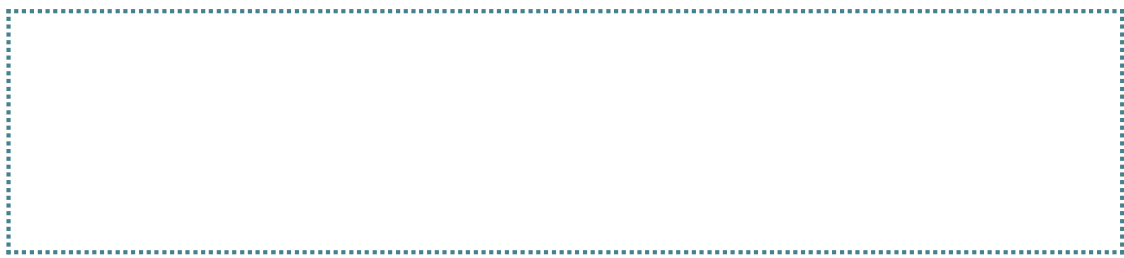
My discussion questions (some examples are included)

- Why is if helpful to draw the net of a box?
- Did you get different answers? Why?

    Write your own discussion questions here:

- _
  _
  _

- _
  _
  _

Productive discussion	This section gives you examples of prompts, cues and questions that you may ask students during or at the end of the problem solving process.

Before you continue, please watch:

[Communication in the Teaching and Learning of Math](#)
More Math 192 Series Videos:
(www.math.uci.edu/mathceo/teachingvideos.php)

- **Assessing Students work**

Here is a differentiated way to choose questions for students or groups, depending on how they progress in the task. There are 4 possible levels of progress:

1 - Little progress
2 - Some progress
3 - Substantial progress
4 - Task accomplished

In what follows, we describe each.

1 - Little progress

- Representing: Represents part of a 3D object in 2D.
- Analysing: Draws some faces correctly, but does not join them up appropriately.
- Interpreting: Cannot interpret own drawing in terms of 3D box.
- Communicating and reflecting: Clear drawing made, with inaccuracies and/or omissions.
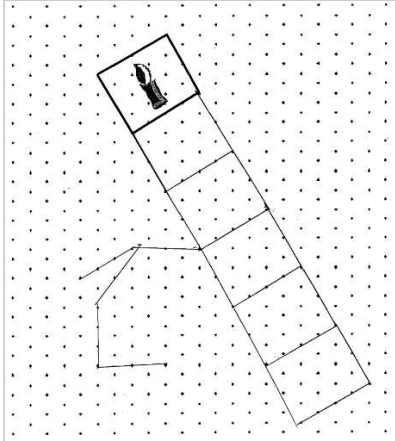
Sample response: Tim
Tim appears to understand what he has to do, and has tried to draw the 6 sides of the box so that they will fold into a ring. These are in the wrong positions, however. He appears to have given up when trying to draw a hexagonal top.

Questions for Tim:Tim could be encouraged to improve his response by asking the following questions:
•*What would your box look like with the faces you have drawn?*
•*Which side of the rectangle should the hexagon base be connected to?*
•*How can you reorganise your design to more closely match the box required?*

2 - Some progress

- Representing: Represents most of a 3D object in 2D.
- Analysing: Draws most faces correctly, joins some up correctly, but there are several omissions and inaccuracies.
- Interpreting: After drawing the net, tries to imagine it folded up again. Draws candles and glue flaps but in wrong orientations/ positions.
- Communicating and reflecting: Clearly draws most of the faces of the box, but with inaccuracies and/or omissions.
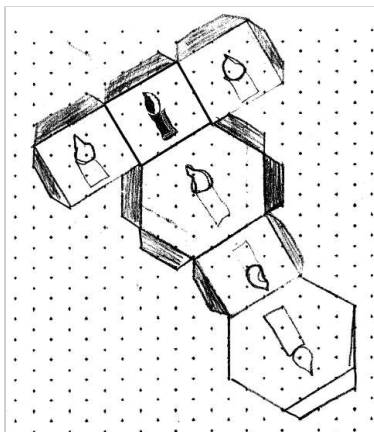
Sample response: Shonah

Shonah has drawn only four of the rectangular sides, but these are the correct size and position. She has also tried to draw the hexagonal base and top in their correct positions, but these are not regular. She has drawn most of the candles in their correct orientations (but not on the lid!). Most of her flaps are correct, but there is a flap where the thumb hole should be.

Questions for Shonah:
Shonah could be encouraged to improve her response by asking the following questions:

•The hexagon that forms the base should be regular How can you adjust the shape of your hexagons to achieve this?
•How many rectangular sides must the box have?
•You have been asked to provide a thumb hole. Where would this go?

3 - Substantial progress

- Representing: Represents a 3D object in 2D.
- Analysing:Draws a net accurately, including the correct number of faces, joined appropriately.
- Interpreting: After drawing the net, imagines it folded up again. Locates some glue flaps and lid flaps, but may not distinguish them. Candle picture may be omitted or in wrong orientation on some faces.
- Communicating and reflecting: Clearly draws and labels the box design.
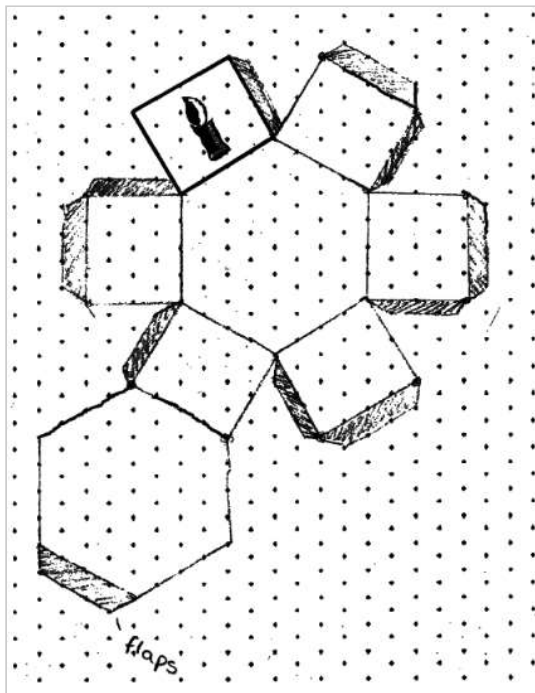
Sample response: Colin

Colin has correctly drawn the base and lid hexagons, and all the rectangular sides of the box in their correct orientations. The candles have not been shown. All but one of the flaps are in the correct place, but Colin has not distinguished between those that are to be glued and those that are not.

Questions for Colin

Colin could be encouraged to improve his response by asking the following questions:

•In which direction would you draw the candles to ensure they will be pointing in the correct direction on the box?
•You have drawn some flaps. What are these to be used for? Which are which?
•Can you complete the box to satisfy all the specifications?

4 - Task accomplished

- Representing: Represents a 3D object in 2D.
- Analysing: Draws a net accurately, including the correct number of faces, joined appropriately.
- Interpreting: Locates some glue flaps and lid flaps, and distinguishes between them. Ensures that these do not interfere with one another. Candle picture in correct orientation on almost all faces.
- Communicating and reflecting: The box design is clearly drawn and labelled.
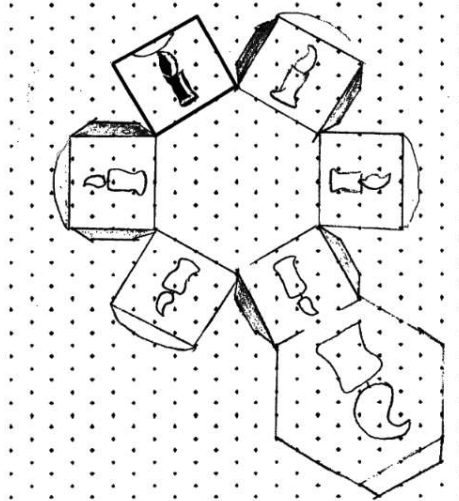
Sample response: Beth
This is an almost completely correct solution, showing the faces and flaps the correct size and orientation. The only error is that the flame on the candle on the lid does not point towards the hinge.

Probing questions for Beth:
Beth could be encouraged to improve her response by asking the following probing questions:
• *Can you imagine the box assembled from your net? Are all the sides present and the candles pointing in the correct direction?*

- **If some students are stuck and cannot begin to make progress**
  - ○ You can show students a net (that is, a "flat version") of a pyramid, for example:
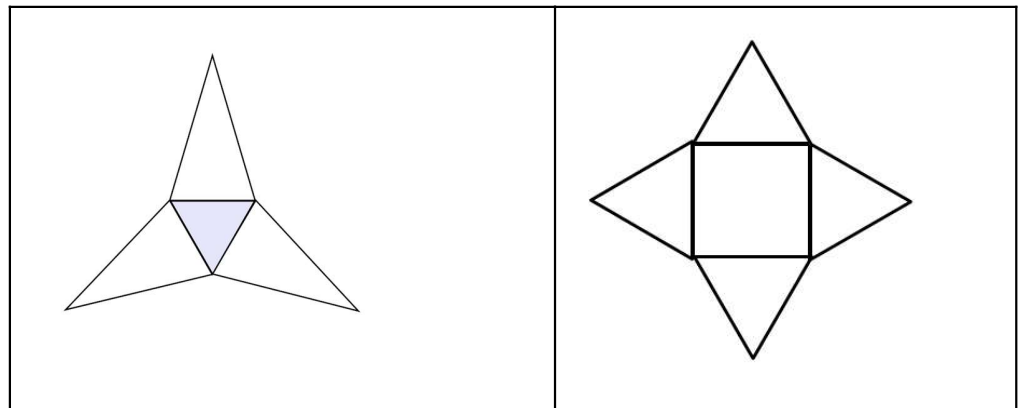
After that, ask them to describe which solid (or "3D-figure" would result in folding these flat pictures.

- ■ This may help students visualize the task in a much simpler case.

| Teaching tips	- Encourage students to keep trying throughout the task. They should not give up, even if somehow they seem to be getting stuck. - Remind students to "imagine" folding the net at any time to see if that helps them come up with new strategies.

Mentor reflection	Based on the 4 levels of progress described above, complete the following formative assessment rubric (summarize content in key words, no need to copy the exact words. Also, create your own question):

	1 - Little progress	2 - Some progress	3 - Substantial progress	4 - Task accomplished
Repre senting				
Analy sing				
Interpre ting				
Commu nicating & reflec Ting				
My question for students				

Use this rubric as preparation for your teaching
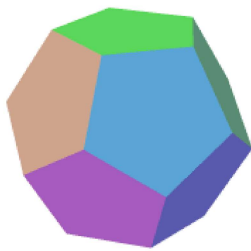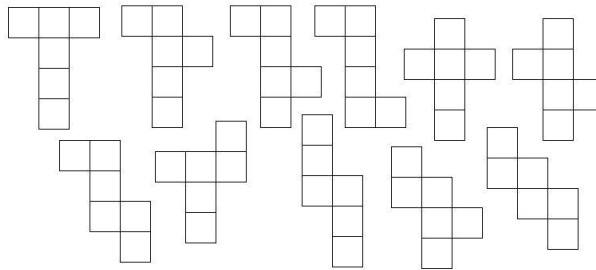
Going deeper (optional)
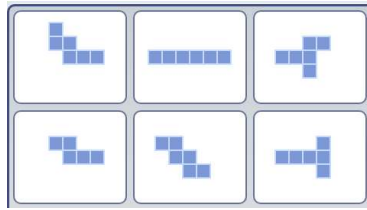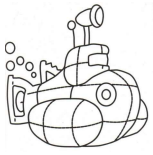
A Net Challenge

For the going deeper, there is a printed challenge (which is not part of the Student Workbook but you will have copies available), where there are different nets, and ask them to select which ones are the nets of a cube (in other words, which ones, when folded, produce a cube).

Solutions:

B Take home (if you still want to challenge students):
You can ask them to draw a net (flat version) of a Dodecahedron. You can describe it in words to them: "In the same way a regular die has 6 square faces, a dodecahedron has 12 faces, and they are identical pentagons. The pentagons are regular, that is, they have the same sides and angles.
You can leave this task as a home project.

Solutions (Candle Box)

See also: http://map.mathshell.org/download.php?fileid=1147

🐵 One solution (note that the orientation of the candles matters).