2

TEACHING MANUAL

PATTERNS

FALL 2018 MEETING 2

OCTOBER 9-10

Contents

www.math.uci.edu/mathceo

2018 UCI MATH CEO COMMUNITY EDUCATIONAL OUTREACH.
UNIVERSITY OF CALIFORNIA AT IRVINE

Teaching Manual PDF

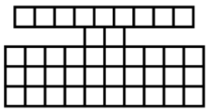
Workbook PDF

Meeting 2 (Oct. 9-10)

- Tuesday 9:00 AM - 9:50 AM: October 2 (UCI Week 1)
  - Place: **UCI** NS 2 1201 (Marco Forester comes)
- Tuesday 2:45 PM - 3:45 PM: October 2 (UCI Week 1)
  - Place: **SANTA ANA**: Carr Intermediate School
- Wednesday 2:00 PM - 3:45 PM (UCI Week 1):
  - Place 1: **UCI,** NS2 1201 (Lathrop comes)
  - Place 2: **UCI,** ALP 2600 : new Anteater Learning Pavillon building (Villa comes)

Tuesday 10/09 (50+ minutes)	Wednesday 10/10 (80+ minutes)

• **Activity 1**: 25 minutes
• **Activity 3 (Research Survey)**: 25 minutes
  ○ Start at 9:30
 	
Skip Activity 2
 	
Note: David Wych will be giving an ongoing CRASH course from 8:45 - 9:00 on Tuesdays (just before de 9:00 AM meeting at NS2 1201)
	• **Activity 1**: 45 minutes
	• **Activity 2**: 15 minutes
	  ○ Only start this activity if time is 3:00 or earlier
	• **Activity 2 (Research survey)**: 30 minutes
	  ○ Start at 3:15



Case 5

Activity 1: SKIP PATTERN

Time: 45 minutes

https://bhi61nm2cr3mkdgk1dtaov18-wpengine.netdna-ssl.com/wp-content/uploads/2017/07/9-12-WIM-3-Skip-Pattern.pdf

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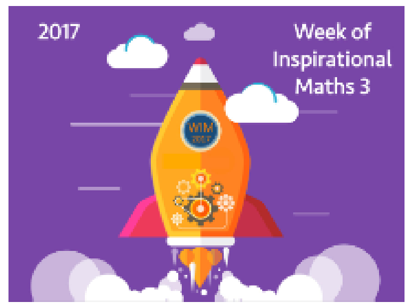
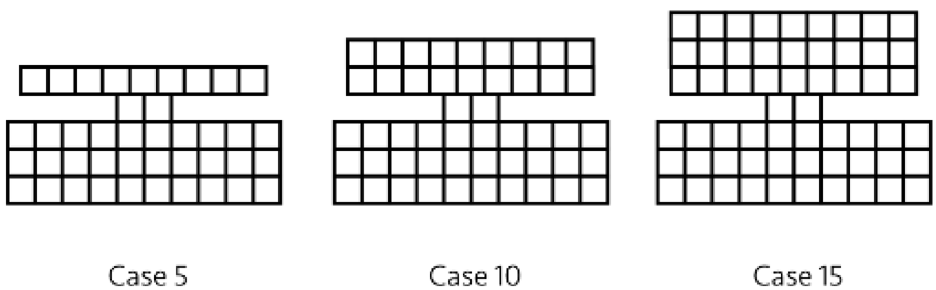
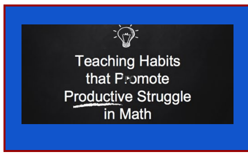


Skip Pattern
Handout

How do you see the pattern growing?



What would case 45 look like?
What would case 8 look like?
What would case -5 look like?
How many squares would be in case 100?
How many squares would be in any case?

Description	This activity requires students to work together to make sense of a visual pattern. There is space in this activity for students to experience multiple ways of seeing patterns and working to describe and explain what they see. This pattern is different from other patterns they may have seen because this visual shows every fifth case. This may be confusing to students at first. Give them freedom to explore the patterns in the different ways they choose.
Materials	• Table templates • Linear graph templates
Set up	• If you want, spent the first 5 minutes describing visual patterns that grow in basic forms such as: xxx, xxxxxx, xxxxxxxxx (3n) ; xxxx, xxxxx, xxxxxxx (3+2n), etc. If you only have little time, skip this part. • Have students start working individually in the pattern. As time progresses, they may feel the need to check work with other students or with you. Don't reveal any answers at the beginning.  Habits of productive struggle in math **Productive struggle** is the kind of effortful learning that develops grit and creative problem solving.
My solution	In this space, write your solution to the problem (working out details, not just the final answers). Use as many visual representations as possible! Also, write discussion questions: these are questions that help students, at the end, consolidate the math learning. **My solution** 

My discussion questions (some examples are included)

- What was the most interesting part of trying this problem on your own?
- Where do you see patterns growing in your school or home? Describe some.

Write your own discussion questions here:

- ____
____
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- ____
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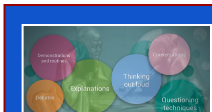
Productive discussion

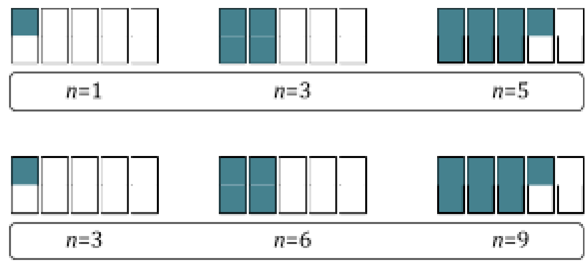
This section gives you examples of prompts, cues and questions that you may ask students during or at the end of the problem solving process.

Before you continue, please watch:



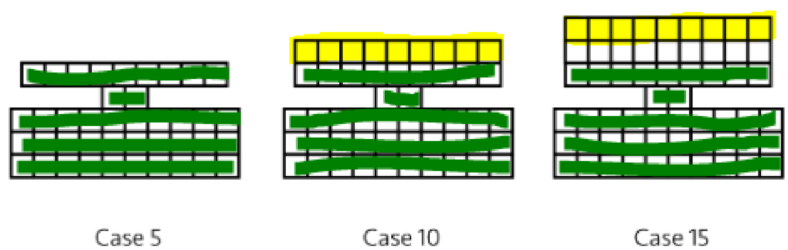
[Communication in the Teaching and Learning of Math](www.math.uci.edu/mathceo/teachingvideos.php)
More Math 192 Series Videos:
(www.math.uci.edu/mathceo/teachingvideos.php)

- **If some students are stuck and cannot begin to make progress**
    - *"Have you seen other types of patterns with numbers or shapes? Give me an example!"* (Have the student draw it for you)
    - *"Consider these two patterns, for example"* (show them)



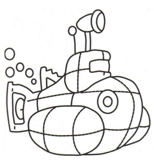
"How do they compare? Which grows faster"
        - Students may not be aware that the "n" is important to determine how fast the pattern is growing. So a pattern has a "pace" in this problem.

- **Providing scaffolding**
    - *"Draw, in different colors, things that you notice in the pattern".*

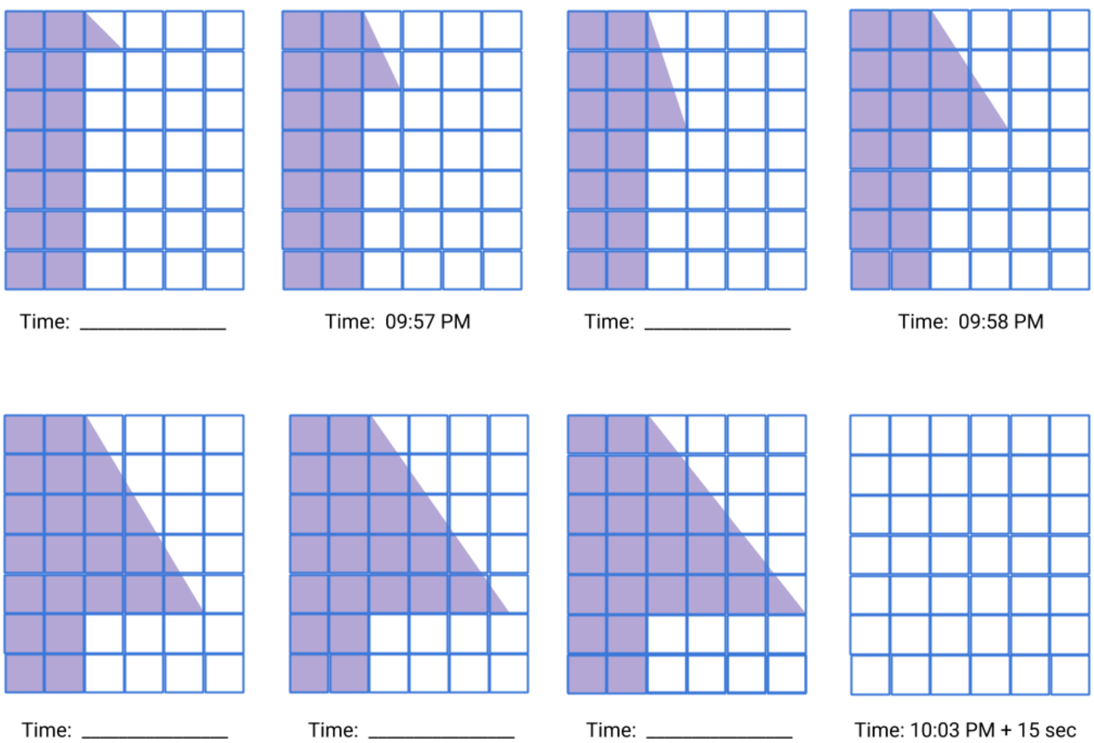


    - *"Draw a new case of the pattern!".*
    - *How would you apply ratios to approach this problem?*
    - *I see that you have some work done. Let's use a table to organize it!*
        - Be careful when providing scaffolding. Do not reveal the answer. Rather have them focus on aspects that can help them come up with productive ways to come up with solutions.

- **Making students check their work**
    - *"Can we check the change from n=5 to n=15 and from n=15 to n=45? Do you see anything odd?"*
    - *"What was your method for getting the pattern at n=8? Explain us!"*
    "If I apply your logic to the case going from n=5 to n=10, what happens?"
        - Positive language, non-judgemental, but critical in a good way. It's important to inspect the process and not just say that the answer is wrong and correct it (it is very tempting to point out the

<table>
<tr><td></td><td>mistake. If you spot it, think on which thinking questions to ask so that your student can spot it too).</td></tr>
<tr><td>Going deeper (optional)
</td><td>Suppose that the area in the sequence grows at constant rate (with respect to time). You need to fill out the times for each frame, and draw the region that corresponds to the last frame (for time 10 hours, 03 minutes, 15 seconds). Justify your answers.

Notes: You may encourage kids to use a table to be able to record the area vs time and check that the area indeed grows at a constant pace.


</td></tr>
<tr><td>Teaching tips</td><td>This problem provides a good opportunity to making sense of unit rates of change. At some point you want students to be able to know that the unit change is 9/5; that is, when n increases by 1, the number of squares (or the area) increases by 9/5.
There are several “roads” to get to that point, and different students will get to that in different times of the process. Some will want to use this as a tool to solve the problem, others will solve the problem and then should conclude that 9/5 is somehow the “core” quantity in the problem. Help those kids reach to that point.
Some sentence stems that might help making sense of the problem:
At n=5, there are ____ squares
At n=10, there are ____ squares
If n grows by 10, the number of squares increases by ____.</td></tr>
</table>

If n grows by 5, the number of squares increases by ____.
If n grows by 1, the number of squares increases by ____.
Extra: If *n* grows by ____, the number of squares increases by 1.

- Since the fractions involved in the problem are fifths and 0.2 = 1/5, this problem gives a good opportunity to strengthen the connection between decimals and fractions. The problem can help students use numerical reasoning to see how, for example 4/5 = 0.8 (4/5 is 1/5 shy of 1, so 4/5 =1 - 0.2 = 0.8).

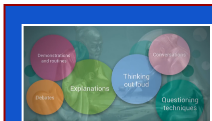
Reflection tasks


Mentor reflection: Come up with 2 “funneling questions” for this task. Funneling questions greatly limit the student opportunities to think and explore. After each funneling question, turn it into one or more focusing questions.

Before you continue, please watch and read:

1) Questioning Revisited: Funneling vs. Focusing
https://www.mydigitalchalkboard.org/portal/default/Content/Viewer/Content?action=2&scId=515243&sciId=19227

2)



Asking mathematical questions
More Math 192 Series Videos:
(www.math.uci.edu/mathceo/teachingvideos.php)

3) Learning to Ask Questions that Engage Students and Deepen Understanding
http://www.wismath.org/Resources/Documents/Annual%20Conference/138BRitsema-Learning%20to%20Ask%20Questions%20PP.pdf

Funneling and focusing questions

Funneling Question 1:

- ____
____
____

I will turn this into one of more focusing questions, as follows:

- ____
____
____

Funneling Question 2:

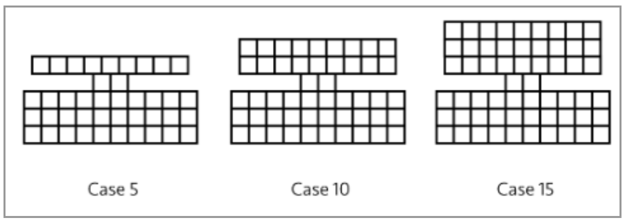
- ____

	____ ____ I will turn this into one of more focusing questions, as follows: • ____ ____ ____

Solutions (Skip Pattern)

See also:
https://bhi61nm2cr3mkdgk1dtaov18-wpengine.netdna-ssl.com/wp-content/uploads/2017/07/9-12-WIM-3-Skip-Pattern.pdf
Source: YouCubed

1) A solution using proportional thinking (table of values):



Let's make a story: suppose I am in an helicopter n is time in minutes, and the picture shows my altitude Y in feet. Then:

(From the first and last picture): In 10 days that passed, my position increased by (32+27) – (32+18) = 18.

So I will conjecture that in 5 days (half of 10), I increase my position by 9 (half of 18). Let's check that:

- From Case 5 to Case 10: it works!
- From Case 10 to Case 15: it works!

This also means that if *n* gets smaller, Y also gets smaller.

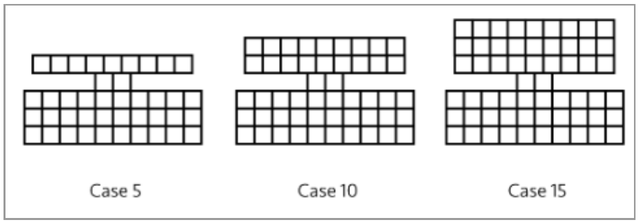
So, we discover that the change is proportional. Using "fifths", we can create a table of values for Y such as this one, where n takes other values (not just multiples of 5). A key observation is that a change of +1 in n yields a change of +9/5 = 1.8 in Y.

n	-5	0	1	2	3	4	5	7.5	10	15	20	30	40	50
Y	23	32	33.8	35.6	37.4	39.2	41	45.5	50	59	68	86	104	122

From this we can answer the different problems: At n=5, there are Y = 41 squares. So, computing always the change from 5, we can say:

- *n*=45: **113** (9 more than the Y for 40, so 104 +9 = 113)
- *n*=8: **46.6** (8 is 2 less than 10, so the Y we are looking for is 50–2(1.8) = 50–3.6 = 46.6.
- *n*= –5: **23** (we see from the table that moving 5 backwards in the the values of n cause the Y values to also move 9 backwards; 41–9–9 = 41-20+2=23) (A good idea is to go back to the helicopter analogy: going back in time means going down in altitude!)
- *n*=100: **232**.
- General case: Y = [9x(n)]/5 + 32 = (9/5)*n* + 32

2)  A solution using proportional thinking (table of changes):



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- From Case 5 to Case 10: it works!
- From Case 10 to Case 15: it works!

So, the change is proportional. Using "fifths", we can create a "table of changes" such as this one, where n takes other values (not just multiples of 5).

Change in n	0	1	2	3	4	5	7.5	10	15	20	30	40	50
Change in "Y"	0	9/5	18/5	27/5	36/5	9	54/5	18	27	36	54	73	92

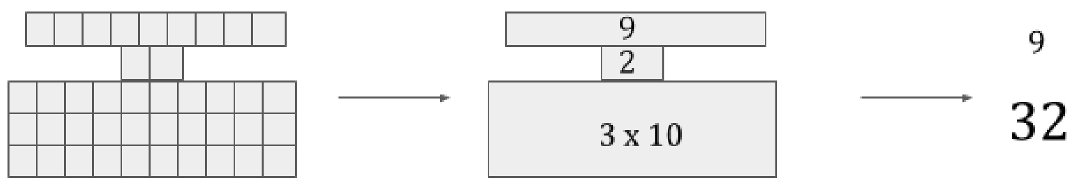
From this we can answer the different problems: At n=5, there are Y = 41 squares. So, computing always the change from 5, we can say:

- *n*=45: change of 40 → Y = 41+8(9) = 41 + 72 = 113.
- *n*=8: change of 3 → Y = 41 + 3(9/5) = 41 + 15/5 + 3/5 =46.6
- *n*=–5: change of –10 → Y = 41 – 18 = 23 (here there should be a discussion about why we subtract 18 instead of adding it. A good idea is to go back to the helicopter analogy: going back in time means going down!)
- *n*=100: change of 95 → Y = 41 + 19(9) = 41+190–19 = 40+1 + 200–10 –20+1=232.
- General case: Y = [9x(n)]/5 + 32 = (9/5)*n* + 32

Note if we add the constant 32 to the Y-values in the table above, we obtain the table for Y-values (asking why would be a good question for students):

n	0	1	2	3	4	5	7.5	10	15	20	30	40	50
"Y"	0+32	9/5 +32	18/5 +32	27/5 +32	36/5 +32	9 +32	54/5 +32	18 +32	27 +32	36 +32	54 +32	73+ 32	92 +32

3) Reasoning out the pattern:



9 x 1	9 x 2	9 x 3	9 x 4
32	32	32	32
Case 5	Case 10	Case 15	Case 20

9 x 1	9 x 2	9 x 3	9 x 4		9 x M
32	32	32	32	...	32
Case 5x1	Case 5x2	Case 5x3	Case 5x4		Case N = 5xM

We first identify common numerical parts to all three graphs: there is a 30, a 2 and a bunch of 9's. Then we remove the pictures and work with numbers only, to gradually get to:

$Case\ N = 5 \times M : \ 32 + 9 \times M$.

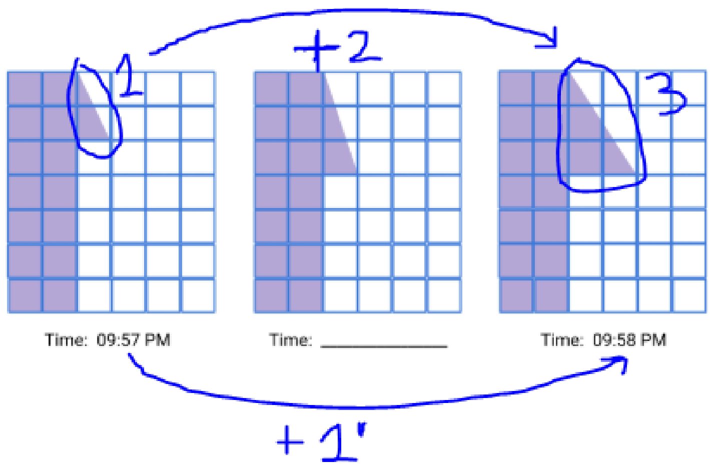
We can stay there, or using that N=5M, get

$Case\ N : \ 32 + 9 \times (N/5)$.

From that general formula, the answers can be answered by substituting N. But first, you can ask students to check that their formula is indeed working!

- Example: N=10, $Case\ 10 : \ 32 + 9 \times (10/5) = 32 + 9 \times 2$ (which matches the picture).

Going Deeper



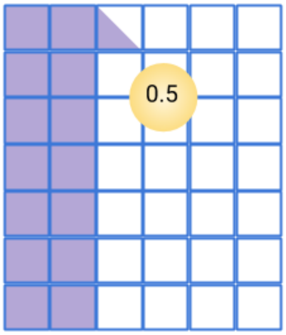
From 9:57 to 9:58 PM, we see that change of area is 3-1=2 (using areas of triangles), and so the area grows 2 every 60 seconds. From there you may make a table or graph, for example... So the area grows 1/2 every 15 seconds.

So the first time is 9:56 and 45 seconds (which is 15 seconds less than 09:57).
Using this type of reasoning, the other times can be filled out.

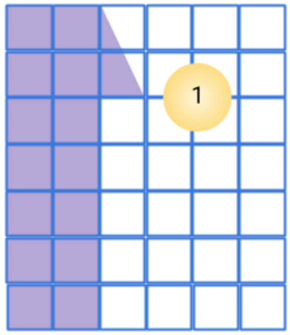
						(1 min)		(2 min)	(5 min)
Change in Time (seconds)	5	10	15	30	45	60	90	120	300
Change in area (square units)	1/6	1/3	1/2 (0.5)	1	3/2 (1.5)	2	3	4	10

The following useful table relates the area of the triangle with the time past 9:57 PM

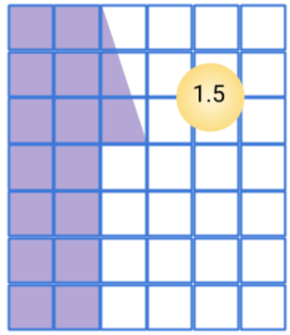
A (units)	1 $\frac{1}{2}$(1/2)x2	1.5 $\frac{1}{2}$(1/2)x3	3 $\frac{1}{2}$(2)x3	7.5 $\frac{1}{2}$(3)x5	8.75 $\frac{1}{2}$(1/2)x2	10 $\frac{1}{2}$(1/2)x2	13.5 $\frac{1}{2}$(1/2)x2
T (m : s)	0 : 30	0 : 45	1 : 30	3 : 45	4 : 22.5	5 : 00	6 : 45



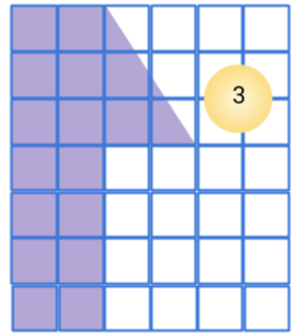
Time: 09 : 56 : 45



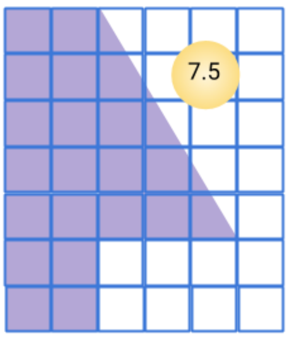
Time: 09:57 PM



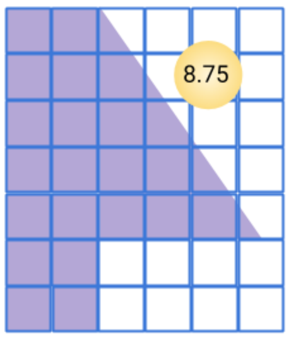
Time: 09 : 57 : 15



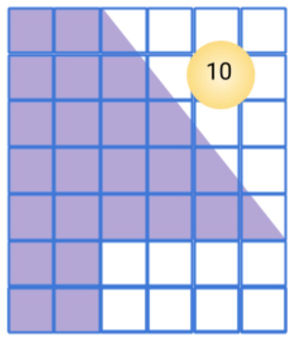
Time: 09 : 58 : 00



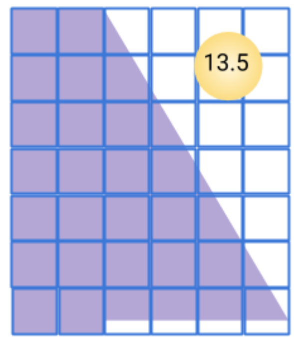
Time: 10 : 00 : 15



Time: 10 : 00 : 52.5



Time: 10 : 01 : 30



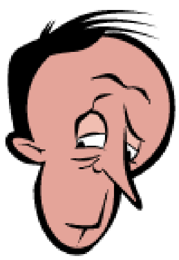
Time: 10 : 03 : 15

Activity 2: HOT UNDER THE COLLAR

Time: 15 minutes
http://map.mathshell.org/tasks.php?unit=ME04&collection=9

HOT UNDER THE COLLAR

John and Anne are discussing how they change temperatures in degrees Celsius into degrees Fahrenheit.



The accurate way is to:
multiply the Celsius figure by 9,
then divide by 5, then add 32.

John

Anne

I have an easier method you can do in your head.
I double the Celsius figure then add 30.
That is near enough for most purposes.

1. If the temperature is 20° C, what is this in Fahrenheit?
   How far out will Anne be, if she uses her method?

2. For what temperatures does Anne's method give an answer that is too high?

Description	In this task, two characters give two sequences of steps to convert temperatures from Celsius to Fahrenheit: one is exact, and the other claims to be a "good" approximation. Students need to make sense of each method and compare them with concrete examples, and then determine thresholds for when is the approximation procedure reasonable.
Materials	None
Set up	Have students work in groups of 2-3. Encourage discussions within the small groups.
My solution	In this space, write your solution to the problem (working out details, not just the final answers). Use as many visual representations as possible! Also, write discussion questions: these are questions that help students, at the end, consolidate the math learning. **My solution** 

<table>
<tr><td></td><td>
 My discussion questions (some examples are included)

Can we use the method of first adding 32 and then multiplying by 9/5 to convert from Celsius to Fahrenheit? What do you think?
What would be a rulte to convert back to Celsius?
Did you see any connection between this problem and the problem before about patterns?

Write your own discussion questions here:

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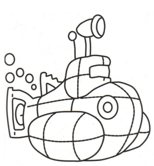
<tr><td>Going deeper (optional)
</td><td>

Certain material can only function if its temperature, in Fahrenheit, satisfies the following condition: if we triple it and subtract 12, it belongs to the following range: (122, 150) (values between 122 and 150).

Find the temperature range, in Celsius, for which the material can function.
</td></tr>
<tr><td>Teaching tips</td><td>

Some kids may not understand why “x2” (multiplying by 2) was chosen in the second method. Ask them why is that. Similarly, ask them why they think +30 is chosen (and for example, int 32 or 34).
If some students want to solve the problem using algebra right away, invite them to complement their explanations with words, and also encourage them to give concrete examples, and to use tables and graphs to make sense of the expressions and expressions.
Depending on your table, some students may be ready to discuss percentual error. You may ask what happens to the percentual error as the value in degrees gets larger and larger.

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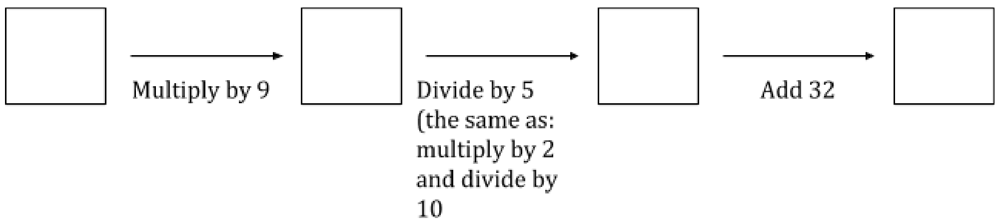
</table>

Solutions (Hot Under the Collar)

See also:
http://map.mathshell.org/download.php?fileid=1155

1) 20 °C: Step 1: multiply by 9, to get 20x9 = (2x9)x10 = 180.
Step 2: Divide by 5 (which is the same as multiplying by 2 and dividing by 10): Obtain 36.
Step 3: Add 32: 36+32 = **68** °**F**.



With the approximation: 20 °C: Step 1: multiply by 2, obtaining 40
Step 2: Add 30, to obtain **70 F**. (Off by 2 °F).

2) We know already that Anne's method gives an answer that is too high at 20 °C. Here are two ways to explore when/if Anne's method gives an answer that is too low:

- Method 1 is to choose two other temperatures above and below 20 °C and see if Anne's method produces answers that are too low or too high.
- Method 2 is to solve for *exactly* when their answers are the same, and explore how their methods differ for temperature hotter or colder than this.

Method 1: Have students choose (or you choose, if you want to guide them) two other temperatures to test. For instance **5** °**C** and **30** °**C** (calculated like above):

5 °**C** → **John:** 5 * 9 / 5 + 32 = **41** °**F;** Anne: 5 * 2 + 30 = **40** °**F** → **Anne is now too low (off by 1** °**F)**
35 °**C** → **John**: 30 * 9 / 5 + 32 = **86** °**F**; Anne: 30 * 2 + 30 = **90** °**F** → **Anne is too high again (off by 4** °**F)**

This leads to…

Method 2: (*hopefully the two guesses showed that at some points Anne's method is too low, at others, too high -- so there must be some point where they are the same*) Solve for the temperature in which they will give the same answer, and talk about what happens for temperatures hotter or colder:

(9/5)*x + 32 = 2*x + 30 → [subtract 30 from both sides] → (9/5) + 2 = 2x → [multiply both sides by 5 to get 5 off the denominator on the left] → 9*x + 10 = 10*x → [subtract 9x from both sides] → **10 = x**

The two methods will give the same result at 10 °C -- Anne's is too low below this point, and too high above this point (now is a good time to ask *why*).