1

TEACHING MANUAL

MEET & MATH

FALL 2018 MEETING 1

OCTOBER 3

Contents

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1) ICE CREAM

2) 25% SALE

www.math.uci.edu/mathceo

2018 UCI MATH CEO COMMUNITY EDUCATIONAL OUTREACH.
UNIVERSITY OF CALIFORNIA AT IRVINE

(MONDAY COPY)

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Meeting 1: Meet & Math! (10/03)

- Tuesday 9:00 AM - 9:50 AM: October 2 (UCI Week 1)
  - Place: **UCI** NS 2 1201 (Marco Forester comes)
- Tuesday 2:45 PM - 3:45 PM: October 2 (UCI Week 1)
  - Place: **SANTA ANA**: Carr Intermediate School
- Wednesday: October 3 (UCI Week 1):
  - Place 1: **UCI,** NS2 1201 (Lathrop comes)
  - Place 2: **UCI,** ALP 2600 : new Anteater Learning Pavillon building (Villa comes)

Tuesday (50+ minutes)	Wednesday 10/03 (80+ minutes)
• **Activity 0**: 30 minutes • **Activity 1**: 20 minutes	• **Activity 0**: 30 minutes (ok if 40 minutes, but no more) • **Activity 1**: 30 minutes • **Activity 2**: 20 minutes *(You may skip Activity 2 and spend all time in Activities 0 and 1)*

Activity 0: MEET YOU MENTOR!

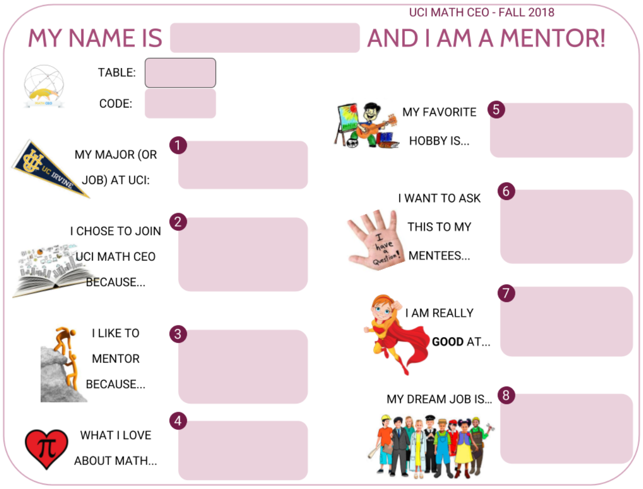
| 30’ |

This activity has the goals of breaking the ice between mentors and students, and promoting interaction and confidence between Students and Mentors. It is also an opportunity for students to practice communication skills.

The activity has 2 parts. In part A “Introduction” (15 minutes), everyone will fill out an About Me form, to be collected. In part B “Unique and Shared” (15 minutes), students and mentors play a short ice-breaking game.

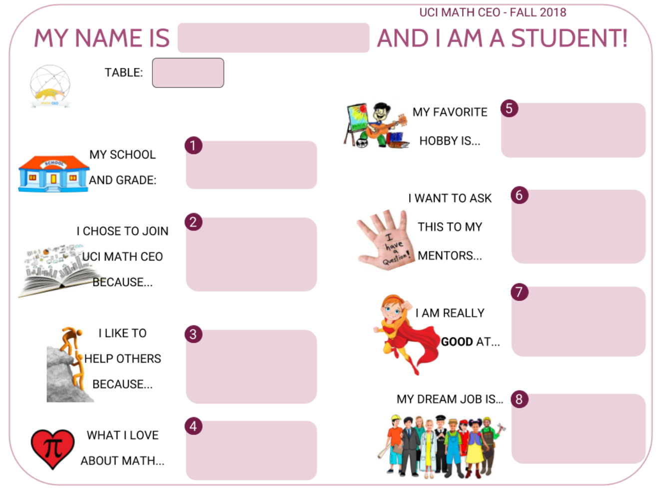
A) Introduction

- Everyone, including the mentors, introduce themselves (2 minutes).
- About Me (8 minutes): Students and Mentors fill out the About Me FORM. You don’t need to print them, we will have it for you at the meeting.
  - Leader mentors: Collect both mentor and student answers to put them back into the team folder. Use a clip.
- Students and mentors share something interesting from their survey, for example the dream or the person they admire the most (5 minutes).



UCI MATH CEO - FALL 2018
MY NAME IS AND I AM A MENTOR!
TABLE:
CODE:
1 MY MAJOR (OR JOB) AT UCI:
2 I CHOSE TO JOIN UCI MATH CEO BECAUSE...
3 I LIKE TO MENTOR BECAUSE...
4 WHAT I LOVE ABOUT MATH...
5 MY FAVORITE HOBBY IS...
6 I WANT TO ASK THIS TO MY MENTEES...
7 I AM REALLY GOOD AT...
8 MY DREAM JOB IS...

MENTOR FORM



UCI MATH CEO - FALL 2018
MY NAME IS AND I AM A STUDENT!
TABLE:
1 MY SCHOOL AND GRADE:
2 I CHOSE TO JOIN UCI MATH CEO BECAUSE...
3 I LIKE TO HELP OTHERS BECAUSE...
4 WHAT I LOVE ABOUT MATH...
5 MY FAVORITE HOBBY IS...
6 I WANT TO ASK THIS TO MY MENTORS...
7 I AM REALLY GOOD AT...
8 MY DREAM JOB IS...

STUDENT FORM

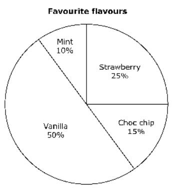
Put **all** the about me forms back into the table folder (including the mentor ones).

B) Unique and shared

https://www.icebreakers.ws/team-building/unique-and-shared.html

The first half is the **Shared part**. Instruct a notetaker for each group to create a list of many traits or qualities that members of the group have in common. Allow about five or six minutes and then have a spokesperson read their list.

The second half is the **Unique part**. On a second sheet of paper have them record Unique traits and qualities; that is, items that only apply to one person in the group. Instruct the group to find at least two unique qualities and strengths per person. Allow another five or six minutes. When time is up, share the unique qualities in one of the following ways: (1) each person can share one of their unique qualities themselves; (2) have each person read the qualities of the person to their right; or (3) have a spokesperson read a quality one at a time, and have the others guess who it was.



Activity 1: ICE CREAM

Using math to plan how to sell ice cream at a school sports event. (Analyze proportional relationships and use them to solve real-world and mathematical problems.)

Time: 30 minutes

http://map.mathshell.org/tasks.php?unit=ME05&collection=9

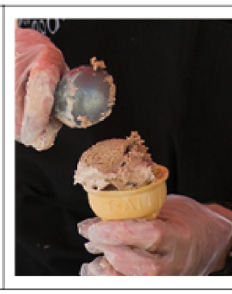
Source: The Mathematics Assessment Project, an initiative by the Bill & Melinda Gates Foundation.

ICE CREAM

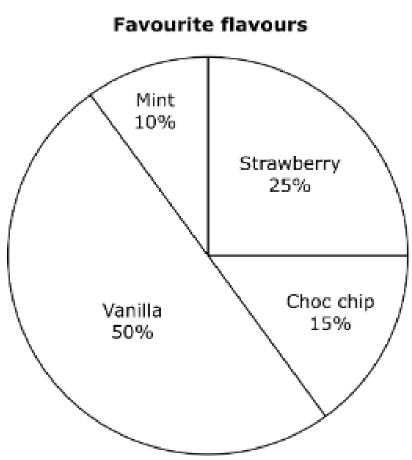
You are planning to make and sell ice creams at a school sports event.

You expect to make and sell 300 ice creams.

You buy ice cream in 1 litre tubs. Each tub costs $2.
You can fill ten cones from each tub.
Each empty cone costs 5¢
You plan to sell each filled cone for 80¢.



Before buying the ice cream, you survey 60 people to find out what flavours they like. Here are the results of the survey:



1. Work out the quantities you need to buy and their costs.
2. How much profit do you expect to make on the day?

Show all your reasoning clearly.

Description	In this task, students are given data on a selling ice cream project, namely: Number of ice creams to sell (300), costs and relations between containers (1 lt tubs, each costs \$2, each is worth 10 cones; each empty cone costs 5¢) and the unit price to sell (each cone for 80¢). Additional details for flavor distribution (percentages) are given in a pie chart. The problem asks to work out through the details of the project and in particular to calculate the profit.
Materials	• Proportionality tables • Double number lines to relate 2 quantities (proportionally). • Triple number lines to relate 3 quantities (proportionally). Using these materials: you can give them to students who are struggling, as a tool to help them make sense of the quantities and relationships. Or, you can give the problem to students who are done as a way to check their answers and gain more understanding. You can also use the materials at the end during discussion, to explain the problem. Make these decisions yourself!
Set-up	• Have students read the problem individually. • Once this is done, ask one or more students to explain the problem **using their own words**. ○ Guide them to be precise in their explanation (but that does not mean using the same words as the statement, in fact, encourage students to use their own words). • Encourage kids to work in groups of 2 or 3. If desired, and depending on your group, you may also do part of the activity all together, leading with questions. If that is the case, make sure to ask questions to all kids, and not just 1 or 2.
My solution	In this space, write your solution to the problem (working out details, not just the final answers). Use as many visual representations as possible! Also, write discussion questions: these are questions that help students, at the end, consolidate the math learning. **My solution** 

My discussion questions (some examples are included)

- If we double the selling price (from 80c to 160c), does this mean that the profit is also doubled? Why?
- Did someone do the problem using mostly ratios or multiplications instead of divisions? How?
- If the cost of each tub is now 90% of the original cost, can someone tell me in words how we find the new profits? Don't calculate it! Just guide me through the math that we would need!

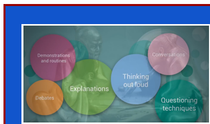
- ____
  ____
  ____

- ____
  ____
  ____

Productive discussion

This section gives you examples of prompts, cues and questions that you may ask students during or at the end of the problem solving process.

Before you continue, please watch:



[Communication in the Teaching and Learning of Math](www.math.uci.edu/mathceo/teachingvideos.php)
More Math 192 Series Videos:
(www.math.uci.edu/mathceo/teachingvideos.php)

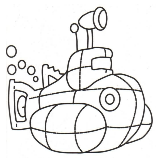
- **If some groups are not able to "start" (overwhelmed)**
  - *"What data is given to you? What would be a good way to represent it?"*
    - Notice that this questions don't "reveal" the solution or method; instead they will help students organize and plan their approaches.

- **If you see two students who seem shy or are working in isolation**
  - *"Hey Alan and Bianca, I see that you are working alone, maybe you want to work together for a while? I think you can learn a lot from each other"*
    - Don't force them to pair up: instead, you should invite them to do so **and provide at least one reason for it.**

- **If you see a student working in isolation who seems quite comfortable figuring out the problem**
  - *"Linda, would you like to present (all or part) of your solution to these students and take questions from them" ; "I see that you have the answers, but it's also important that you can talk and convince others"*
    - This can be especially useful to spark communication skills in **students who do not see themselves as "good communicators"** but are confident in math.

- **Scaffolding / testing for understanding**
  - *"Focus on the case where we only need to sell 1 ice cream cone. How does the problem look like?" (later...) "How about 10 now?"*
  - *"What is your logic for doing this operation?"*
    - For example, subtracting those numbers, doing a particular multiplication or division, etc.) Ask this whether the student has made a mistake or not: if so, use this as an opportunity for students to reflect on their mistake.

- **If you see a wrong solution**
  - *"I'm curious why you got this value. Guide me through it! I want to understand what you were thinking"*
  - *"Why are you saying that this is always the case? I would like to understand"*
    - Notice the positive language, non-judgemental, but critical in a good way. It's important to inspect the process and not just say that the answer is wrong and correct it (which is tempting but will not result in meaningful learning from the student, since you will not reach the "source of the mistake".

<table>
<tr>
<td>Going deeper (optional)
</td>
<td>In this task, we keep the same values as before, except that now students need to decide on the unit price to maximize profits.

During a special day, you plan to sell a brand-new “Greeny” flavor. You surveyed 100 school kids and discovered that:
● If you sell each cone at 80 cents, then 50 kids will buy the cone.
● For every 6 cents that you raise the price of each cone, you will lose 4 clients.

What should be price to gain as much money as possible? This is called “maximizing the profit”: you are making the profit as large as you can.</td>
</tr>
<tr>
<td>Teaching tips</td>
<td>● It’s always a good idea to start the activity with an informal chat with students about the real-world situation that is presented.
○ Ask students if they have participated in things like selling food for profit, and how it went. Or if they haven’t, ask them what they would need in terms of logistics.
○ This warm-up can help students to get engaged in the math.

● This is the first meeting, and so it is very important to set the tone. Make sure that you encourage your students to talk. Some of them will be quite shy at the beginning, but keep encouraging: you will see how little by little the open up (this is a common experience with past mentors, who have always mentioned this). Here is an excellent resource to help you be prepared:

How to Get Students to Talk in Class
https://teachingcommons.stanford.edu/resources/teaching/small-groups-and-discussions/how-get-students-talk-class
Source: teaching commons, Stanford

● When asking questions, make eye contact with students. Listen carefully, with undivided attention. They will care more if they see that you care for their thinking (not just their answers).

● Before starting, clarify the money language involved in the problem: cost, sell at, profit. You can provide sentence stems such as:
○ The cost was ___ and i sold it at 80, so the profit was 13.

● This problem presents a great opportunity to consolidate basic facts about multiplication of decimals with integers. Examples:
○ 5.5 x 10 = 55
○ 2 / 10 = 0.2
○ 5.5 x 30 : first multiply by 10, then by 3! (5.5 x 30 = 55 x 3 = 160).

Make sure you comment on these facts as general rules, not just particular instances of the problem. You may ask your students to tell you these types of rules, using their own words.</td>
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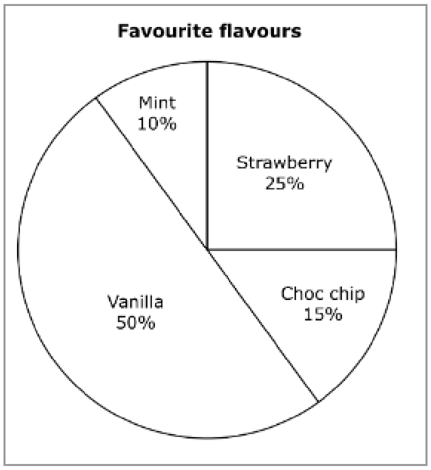
</table>

Solutions (Ice Cream)

See also: http://map.mathshell.org/download.php?fileid=1158
Source: The Mathematics Assessment Project, an initiative by the Bill & Melinda Gates Foundation.

On the flavors



With 300 people (which is 5 times 60), you would expect:

- 150 people who want vanilla
  - 50% of 300 is 150. Note that 60 people were surveyed, but we are "scaling up", and thus projecting that about half of the 300 attendants would want Vanilla.
- 30 people who want mint (amounts to 3 tubs)
  - 10% of 300 is 30
- 75 who want strawberry (amounts to 8 tubs, rounding up)
  - 25% of 300 is (½)(½)(300) = 75
- 45 who want chocolate chip (amounts to 5 tubs, rounding up)
  - 15% of 300 = 30 + "half of 30" = 45. (In general, 15% of a quantify is 1-tenth of a quantity plus half of that 1-tenth.

Students can also make a proportionality table like this to obtain these numbers:

	Vanilla	Mint	Strawberry	Choc chip
Percentage	50%	10%	25%	15%
For 100 people	50	10	25	15
For 300 people	**150**	**30**	**75**	**45**

This means that if we want to "please everyone" according to the sample you would buy 15 + 3 + 8 + 5 = 31 tubes of ice-creams (thus wasting 1 tube). If you are OK with expecting 5 people out of 60 (that is, 25 out of the 300) not getting what they want, and make sure you do not waste any ice-cream, then you will buy 30 tubes.

One should probably choose Option #2: we surveyed 60 people only, and that is a sample... it **only gives you an indication** of what the 300 people who actually show up at the event may want. We are not certain that the numbers worked up will be indeed exact.

On the profit

We will assume that we chose option #2 and bought 30 tubs.

The triple ratio of liters (L) to price (the cost $P) to number of cones (C) is 1:2:10.
Since 300 cones will be sold and 300 is 30 times 10, we need to pay 30×2=60 to buy the ice cream.

Liters	Ice cream cost in dollars	# of cones
1	**2**	**10**
1/10 or 0.1	$0.2 (or 20 cents)	1
10	20	100
30	60	300

The first row was given, and we build the other rows using proportionality

Profits: for each ice cream cone, we spent 20 cents in ice cream (since we spent $2 for each 10 cones), 5 cents in the cone, and we will sell each unit for 80 cents. 80-20-5=55, so the **unit profit** equals to 55 cents per cone.

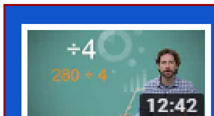
Then: 100 ice cream cones s yield a 55 dollar profit. Then
300 ice cream cones s give a profit of 55 x 3 dollars, so the total profit is
50+50+50 + 5+5+5 = **$165**.

Another way to reason: We spent $60 in ice cream. As far as the cones are concerned, we spent 5 cents per cone ---> 10 cents per 2 cones ---> $1 per 20 cones ---> $5 per 100 cones ---> $15 per 300 cones (all of them).
So the total cost is 60 + 15 = $75.

We sold each cone at 80 cents. So if we multiply 300 x 80 cents, we get the total money. In dollars:

- $300 \times 0.8 = 300 \times (4/5) = (300/5) \times 4 = (600/10) \times 4 = 60 \times 2 \times 2 = 240$
- Or: $300 \times 0.8 = 300 \times 0.1 \times 8 = 30 \times 8 = 240$
- Or: $300 \times 0.8 = 300 \times (1 - 0.2) = 300 - (300 \times 0.2) = 300 - 60 = 240$.

So profit is: $240 - 75 = 240 - 40 - 35 = 200 - 35 = 165$ dollars.



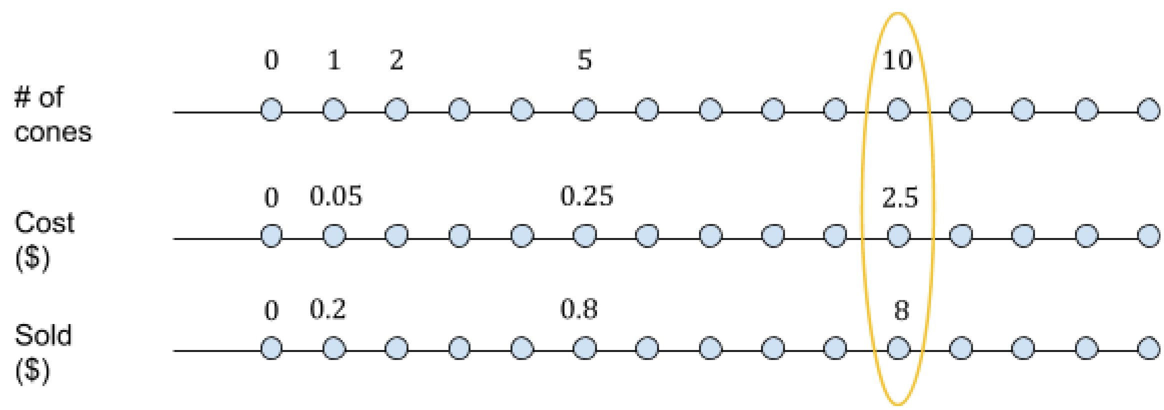
Strategies for multiplication and division

More Math 192 Series Videos: (www.math.uci.edu/mathceo/teachingvideos.php)

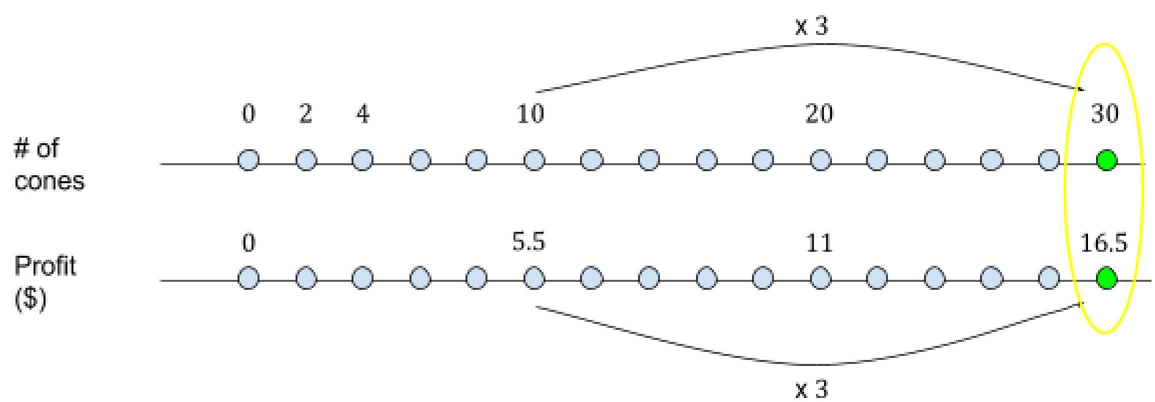
Here is a more complete table:

Liters	Ice cream cost in dollars	Cone cost in dollars	Total cost	Sold at (dollars)	Profit (dollars)	# of cones
1	**2**	0.5	2.5	8	5.5	**10**
1/10 or 0.1	1/5 or 0.2	**0.05**	1/4 or 0.25	**4/5 or 0.8**	0.55	1
10	20	5	25	80	55	100
30	60	15	75	24	165	300

One can also use double or triple number lines (which work like tables for proportionality relations, but are more "visual"):



For example, the triple number diagram shows that for 10 cones, the profit is 8-2.5 = 5.5. This can help us find the profit for 300 cones (multiplying by 30). Notice that dots are ***equally spaced****.*



Or, using a double number line relating # of cones and Profit (directly), we can see by proportional reasoning that 30 cones yield $16.5 in profits, so 300 cones will yield 10 times as many profits, that is, $165.

Mentor reflection

Complete the following table, mentioning Pros and Cons for each representation (in general, not just for the activity):

	Type of mathematical representation		
	Equations	**Double/Triple Number lines**	**Table**
Cool (advantages)			
Not so cool (limitations)			

Solution to Going deeper

During a special day, you plan to sell a brand-new "Greeny" flavor. You surveyed 100 school kids and discovered that:

- *If you sell each cone at 80 cents, then 50 kids will buy the cone.*
- *For every 6 cents that you raise the price of each cone, you will lose 4 clients.*

What should be price to gain as much money as possible? This is called "maximizing the profit": you are making the profit as large as you can.

Unit price	# of Buyers	Profit $
0.74	54	26.46
0.77	52	27.04
0.8	50	27.5
0.83	48	27.84
0.86	46	28.06
0.89	44	28.16
0.92	42	28.14
0.95	40	28
0.98	38	27.74
1.01	36	27.36
1.04	34	26.86
1.07	32	26.24
1.1	30	25.5
1.13	28	24.64
1.16	26	23.66
1.19	24	22.56
1.22	22	21.34

Solution: The first important step is for kids to make sense of the problem, in particular making sense of the statement "*For every 6 cents that you raise the price of each cone, you will lose 4 clients*". In particular, this implies:

- For a unit price of $0.86, there are 46 buyers
- For a unit price of $0.74, there are 54 buyers.

You can provide students with sentence stems that account for this, before generalizing:

a) For a unit price of $0.8, there are _____ buyers
b) For a unit price of $0.86, there are ______ buyers
c) For a unit price of $ _____, there are 58 buyers.

Students can start at X = 0.8, Y = 50 and then try "moving" up or down, so to speak, in X. They will see that as X goes down (if they charge less), the profit seems to start decreasing.

So X should move up, little by little, up to the point where Y reaches a top value. By inspection, this seems to be where X = 0.89 or X = 0.9 even. One can graph Y in terms of X (it's an upside down parabola) or even use calculus for maximizing Y, but this is not the intended solution method at this level. What's important is that kids understand the problem, make sense of the condition, and come up with a nice trial and error method (for example) to get a reasonable answer (in the range [$0.89, $0.91]).

Activity 2: 25% SALE

Time: 20 - 30 minutes

http://map.mathshell.org/download.php?fileid=1042

25% Sale

In a sale, all the prices are reduced by 25%.

1. Julie sees a jacket that cost $32 before the sale.
   How much does it cost in the sale?

   $ ______________________

   Show your calculations.

In the second week of the sale, the prices are reduced by 25% of the previous week's price.
In the third week of the sale, the prices are again reduced by 25% of the previous week's price.
In the fourth week of the sale, the prices are again reduced by 25% of the previous week's price.

2. Julie thinks this will mean that the prices will be reduced to $0 after the four reductions because 4 x 25% = 100%.

   Explain why Julie is wrong.

   ____

   ____

   ____

3. If Julie is able to buy her jacket after the four reductions, how much will she have to pay?

   $ ______________________

Show your calculations.

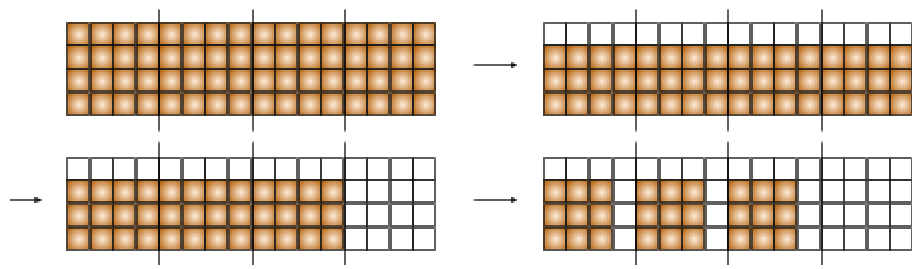
Julie buys her jacket after the four reductions.
What percentage of the original price does she save?

____________________%

Show your calculations

<table>
<tr><td>Description</td><td>In this task, students compute price increase or decrease of items according to a percent change. They also learn to interpret repeated percent decrease and what it means in terms of subtraction or multiplication.

The key takeaway is that repeated percent increase is like multiplying several times by the same “enlarging” or “shrinking” factor, and that each time you multiply again, the enlarging has a bigger absolute impact (or the shrinking has a smaller absolute impact). So the impact is not constant, it changes, and that is the reason why repeating a 25% discount 4 times in a row does not make the original quantity disappear, as each discount is less (in absolute terms).

Examples:
● 64 -- 25% off → 48 -- 25% off → 36 --- 25% off → 27 --- 25% off → 20.25.
● 100 -- 25% off → 75 -- 25% off → 56.25 --- 25% off → 42.1875 --- 25% off → 31.640625.
● </td></tr>
<tr><td>Materials</td><td>● Number strips (divided into 16)
● Calculators (optional, may use a smartphone)</td></tr>
<tr><td>Set-up</td><td>You may keep the same groups as before. But this time, allow for 1-2 minutes of individual work for question 1).</td></tr>
<tr><td>My solution</td><td>In this space, write your solution to the problem (working out details, not just the final answers). Use as many visual representations as possible! Also, write discussion questions: these are questions that help students, at the end, consolidate the math learning.

My solution
</td></tr>
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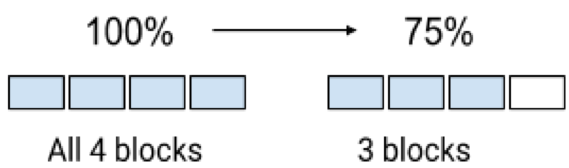
My discussion questions (some examples are included)

- If we double a quantity, then we double it again and then we double it again, I say that it's the same as multiplying by 6. I am lying? Why?
- Did someone do the problem using fractions? Algebra (variables)? Pictures? Show us all!
- ____
- ____

</td>
</tr>
<tr>
<td>Productive discussion</td>
<td>This section gives you examples of prompts, cues and questions that you may ask students during or at the end of the problem solving process.</td>
</tr>
</table>

- **If some groups are not able to "start" (overwhelmed)**
  - *"Suppose that an item costs $100 and we discount 25%. What is the new cost?"*
  - *What is the meaning of taking 25% of a quantity? How can you draw it? How to write it with an equation?*



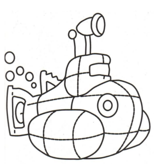
    - Encourage students to draw strips to make sense of the action "remove 25%" and understand that it is the same as "keep 75%".
    - No algebra is needed here, but it can help complement understanding if some of your students are ready.

- **If you see two students who seem shy or are working in isolation**
  - *"Hey Alan and Bianca, I see that you are working alone, maybe you want to work together for a while? I think you can learn a lot from each other"*
    - At this point, you may choose to keep the same groups from the previous activity, but you may choose to form new groups strategically (to prevent some distractions, or to pair students of different mathematical backgrounds).

- **If you see a student working in isolation who seems quite comfortable figuring out the problem**
  - *"Linda, would you like to present (all or part) of your solution to these students and take questions from them" ; "I see that you have the answers, but it's also important that you can talk and convince others".*
  - *I see that you made a very nice diagram. Please share it!*
    - Insist on this, but still be very gentle about it. If the discussion in activity 1 was "dominated" by only a few students, focus on other students. Don't let anyone behind!

- **Scaffolding / testing for understanding**
  - *"Do you know how to take 75% of a quantity?" "Do you understand why this is the same as removing 75%?*
  - *"Ok, so how can you use this discovery to approach the problem"*
  - *"How would you represent your calculations with a strip?"*
    - Whether the student uses pictures, decimals, fractions or algebra (hopefully more than one), your job is to make sure that the student understand the concept and doesn't just know how to apply the procedure.

- **If you see a wrong solution**
  - *"I'm curious why you got this value. Guide me through it! I want to*

<table>
<tr><td></td><td>
understand what you were thinking. How did you apply percentages? Where do I see here the original cost?”
“Why are you saying that this is always the case? I would like to understand”
[notice the positive language, non-judgemental, but critical in a good way. It’s important to inspect the process and not just say that the answer is wrong and correct it (which is tempting but will not result in meaningful learning from the student, since you will not reach the “source of the mistake”]
</td></tr>
<tr><td>Going deeper (optional)
</td><td>
The following task is optional and can provide a new complexity layer, building on the previous one:

Today, a giant is X inches tall (you may use X=640 -concrete-, keep it as X, or discuss both X=640, and X in general in parallel). Each time he whistles, his height increases by 25%.

a) What is the giant’s height after 1 whistle? Illustrate using a number strip (divided into 4).

b) How many whistles total does the giant need in order to triple its height? Estimate the answer before going about solving it.

c) Every day, the giant wakes up with its original height. Suppose that tomorrow, we double the number of whistles used in part b). Do you think that the height gets multiplied by 6, less than 6 or more than 6? Justify your thinking. Then use a calculator to find the answer.
</td></tr>
<tr><td>Teaching tips</td><td>
This activity provides an opportunity to learn the fact that taking 25% is equivalent to dividing by 2, two times. Example:

25% of 14 is 3.5 (14 --> 7 ---> 3.5)
A nice way to challenge students who seem to be grasping the concepts well is to ask them: Ok so 4 “steps” of 25% did not make the value equal to zero. So, how many do we need? Encourage kids to reason in pictures to realize that no matter how many steps, you will never reach $0. Then you could ask them if they think, for example, that after several steps, the value reaches (for example) $1, or even $0.01 (1 cent). You may first do this for the case of 50% reductions.
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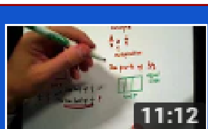
Solutions (25% sale)

See: http://map.mathshell.org/download.php?fileid=1043

1) There are several methods:

a) Using Fractions: 25% reduction is the same as "keeping 75%" 75/100 equals 3/4 which will mean dividing the original number into 4 equal pieces and keep 3 of them. So:

Discount of 25% means taking 75% of 32, which is $\frac{3}{4}(32) = 3 \times \frac{32}{4} = 24$.



Fraction multiplication

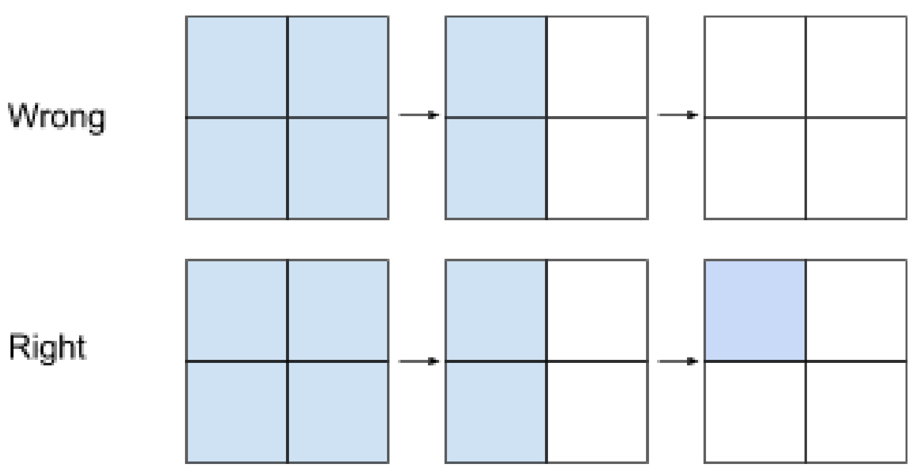
More Math 192 Series Videos: (www.math.uci.edu/mathceo/teachingvideos.php)

b) Using decimals: 0.75 x 32 = 75 times 32, divided by 100:
70 times 32 = 2240 ; 5 times 32 = 160. Add: 2400. Divide by 100: **24**.

2) While it is valuable that the student can solve the problem correctly, what is key here is that they can explain why Julia is wrong. In other words, what is Julia thinking incorrectly when performing the operations. So the students need to "confront" Julia, rather than just saying: *"Look, i got this value, which is not 0, so Julia must be wrong"*. This makes the problem quite compelling.

There are several explanations. Here is an example.

Example: Julia is always taking 25% of the original quantity and she does not realize that the price is being reduced, so the discounts are getting smaller and smaller. A picture can help illustrate my point. I will do it with 50% discount because I think it is easier for me (and for you to follow me):



The first row is wrong in the second step, because 50% was remove, but not 50% of the second quantity, but 50% of the first quantity! And the shirts problem is similar. The second time we take discount, it is now a discount of the already reduced price.

3) We need to start with 32, and each time remove 25% of the quantity we have.

a) One way: perform 25% reduction, four times, one after the other:

- 25% of 32 is 8. So 75% is 32-8 = 24
- 25% of 24 is 6. So 75% is 24-6 = 18
- 25% of 18 is 4.5. So 75% is 18-4.5 = 13.5
- 25% of 13.5 is: (13.5 → 6 + 0.5 + 0.25 ---> 3+0.25 + 0.125 = 3.375)
  So 75% is 13.5 - 3.375 = **10.125**

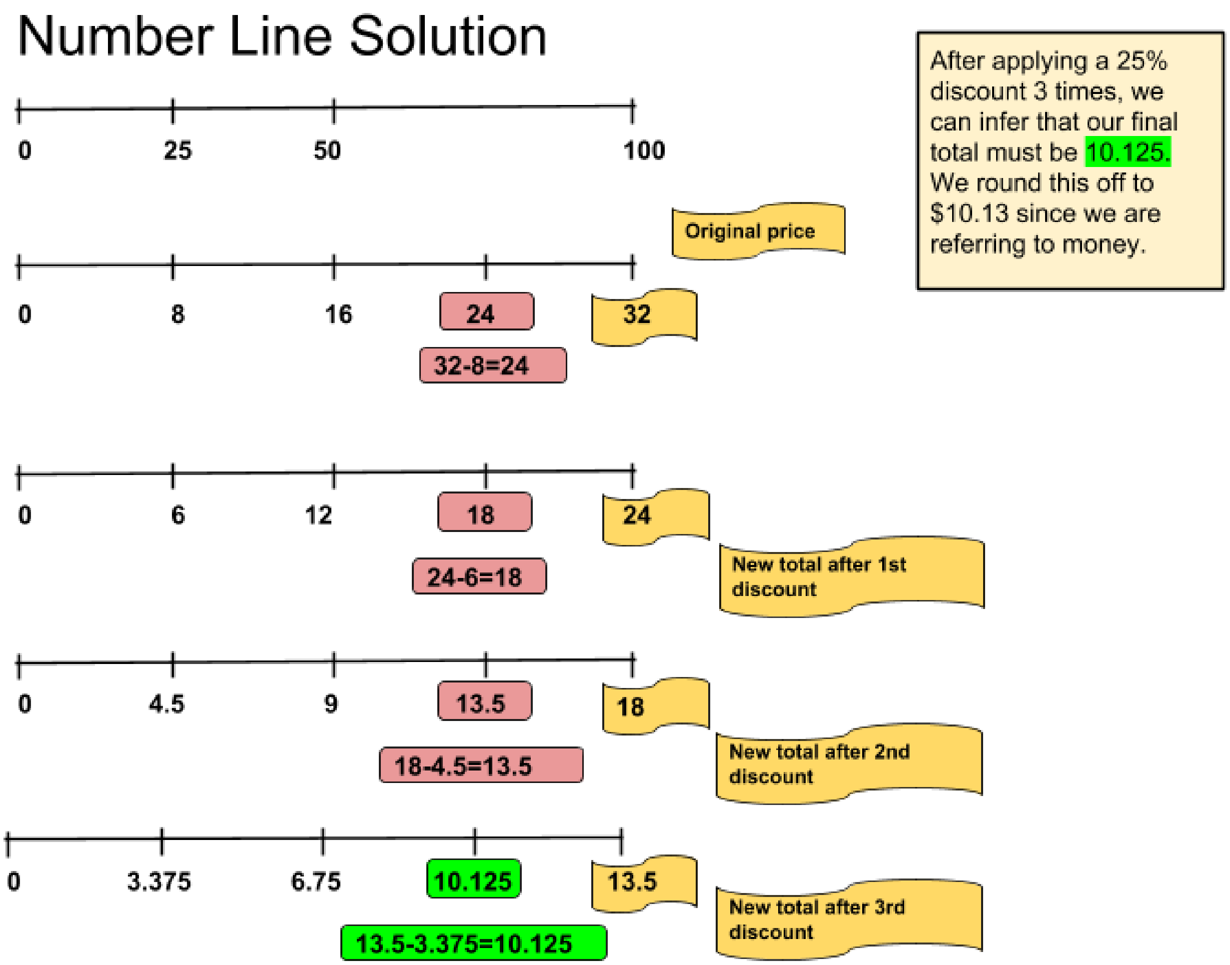
So: 10.12 or 10.13 (since it is money, and we cannot "break" a cent!)

b) Another way using Fractions:
32
---> 3/4 of 32
→ 3/4 of (3/4 of 32)
→ 3/4 of (3/4 of (3/4 of 32))
→ 3/4 of (3/4 of (3/4 of (3/4 of (3/4 of 32)))
The word "of" is a multiplication! So what we have is: $\frac{3\times3\times3\times3}{4\times4\times4\times4}(32)$.
We can cancel some 2's to get: $\frac{3\times3\times3\times3}{2\times4} = \frac{9\times9}{8} = 81 \times \frac{1}{8}$.

0.8 = 0.125. So a good strategy to find the answer would be:

i) 80 x 0.125 = 10 x 8 x 0.125 = 10 x 1 = 1.
ii) 1 x 0.125
Add: 1 + 0.125 = **10.125**
So again: 10.12 or 10.13

c) Another way: using a number line



In this solution, students will see that taking the 25% discount is equivalent to taking a fourth away from the original price. Since the discount is 75% students should iterate this process 3 times (as 25*3=75). Each time they iterate the process, they must update the scale.

 Mentor reflection: In the image above, the first line goes from 0 to 100 and the second line goes from 0 to 32. Suppose that a student sent you a letter asking: “Why is the first line going from 0 to 100 and the second from 0 to 32? I don’t understand what one has to do with the other, can you guide me?”

My response

Dear student: